

Cosmological brane systems in warped spacetime

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Abstract

We discuss the time-dependent brane solutions in higher-dimensional supergravity theories. In the near horizon limits, where the time dependence is negligible, these branes describe warped AdS spacetimes as in the corresponding static solutions. The lower-dimensional cosmological dynamics obtained after compactifications of the higher-dimensional solutions are also examined. The cosmological solutions we have found give the four-dimensional universe with power-law expansion.

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I. INTRODUCTION

This paper is devoted to a study of dynamical solutions of the p -brane in the higher-dimensional supergravity model. We discuss the cosmological models constructed from time-dependent p -branes in which intersect with a Kaluza-Klein monopole, that is, a Taub-NUT with charge M_K . We derive the exact dynamical solutions in the eleven- and ten-dimensional supergravity theory, which can be solved and has been extensively studied [1–23]. We will see that there are still things which are solved about delocalized or partially localized intersecting brane systems. To state our result in ten-dimensional string theory, we will explain a simple T-duality mapping from IIA theory to IIB theory or vice versa in ten dimensions, and dimensional reduction from eleven-dimensional theory to ten-dimensional type II theory. The dynamical solutions in string theory will coincide with intersecting brane solutions, in the form that these have been presented in [24] and further studied in [25]. Mathematically, there are time dependent warp factors with field strengths and scalar field in higher-dimensional spacetime, and the coupling of scalar field takes the same value as the static case. However, it has been known that the time dependence of the warp factor becomes negligible in the near horizon limit. This is a consequence of a lot of works [8, 9, 11, 18] and was used in [21] to obtain precise formulae for partially localized brane systems involving two branes. Studies about the dynamical brane system have suggested that the relation to the supergravity models is not limited to the case of pair intersection for two branes. These are the main properties that we would like to understand. Coming to grips with the dynamics of the four-dimensional universe is the goal of the present paper from a cosmological standpoint.

We will consider solutions with particular couplings of dilaton to the field strengths. In the classical solution of a p -brane, the coupling to dilaton for field strengths includes the parameter N . This is specified in string theory as $N = 4$. Though there are classical solutions for other values of coupling, the solutions in these models are no longer related to D-branes and M-branes. The dilaton coupling is related to the power of the warp factor in the metric and ensures the existence of a classical solution of p -branes in string theory. We know that it is not easy to find a time dependent intersecting brane solution under the assumption of $N \neq 4$ [21].

As in the case of other dynamical brane systems, we assume that the warp factor de-

depends not only on the coordinate of the relative and overall transverse space but also the worldvolume coordinate. In a suitable ansatz for fields, we find that the warp factors arise from field strengths in the time dependent background and then a system composed of n branes can be characterized by n warp factors arising from n field strengths. Unfortunately, there are few solutions in which all harmonic functions depend on time. Many other interesting models contain cosmological solutions, as a result of which they are not such close relatives of supergravity theories if all warp factors in the metric depend on time [19]. These time dependent solutions are still inadequate for explaining cosmological behavior, such as inflation or accelerating expansion of our four-dimensional Universe.

In the present paper we will make this result more transparent and generalize it, although the dynamical intersecting brane solutions involving several kinds of branes have been discussed in [19, 21]. We will take the followings as the starting point. We will construct various explicit intersecting dynamical D-brane, M-brane solutions in eleven and ten dimensions. We give classification of them, and discuss the application of these solutions to cosmology in sec.II - VI. We will study the p -brane on the fiber coordinate of the lens space and pp-wave background, and will obtain the partially localized brane solution involving three branes in terms of Kaluza-Klein reduction or T-duality map. We also provide brief discussions for intersection pairings on the massive supergravity theory, which are presented in V. In section VI, we describe how our Universe could be represented in the present model via an appropriate compactification. We show that there exist no accelerating expansion of our Universe, although the power-law expansion of the universe is possible. To illustrate this, we construct cosmological models of the Dp - $D(p+2)$ -NS5 brane system, which is relevant to type II string theory. We give the classification of the Dp - $D(p+2)$ -NS5 brane, M-brane KK monopole systems and their application to cosmology. Section VII is devoted to conclusion and remarks.

II. DYNAMICAL D3-BRANE SOLUTIONS

In this section, we construct the time dependent brane solution in ten-dimensional IIB theory. These solutions give AdS in warped ten-dimensional spacetime at the near-horizon limit of the intersecting branes. We also discuss the cosmological F1-D2 brane solutions in ten-dimensional IIA theory after performing a T-duality transformation in the time depen-

dent D3-pp wave system.

A. Dynamical D3-KK monopole solution

We first consider the dynamical D3-KK monopole solution of type IIB supergravity. The field equations can be written as

$$R_{MN} = \frac{1}{4 \cdot 4!} F_{MABCD} F_N{}^{ABCD}, \quad (1a)$$

$$d[*F_{(5)}] = 0, \quad (1b)$$

$$F_{(5)} = \pm * F_{(5)}. \quad (1c)$$

Now we will briefly summarize the results for the dynamical D3-KK monopole solution in type IIB supergravity [24].

TABLE I: Dynamical D3-brane and KK-monopole system in the metric (2a). Here \circ denotes the worldvolume coordinate and \bullet denotes the fibre coordinate of the KK-monopole respectively.

	0	1	2	3	4	5	6	7	8	9
D3	\circ	\circ	\circ	\circ						
KK	\circ	\circ	\circ	\circ	\circ	\circ	\bullet			
x^N	t	x^1	x^2	x^3	y^1	y^2	v	z^1	z^2	z^3

The dynamical D3-brane solutions with a single RR 5-form $F_{(5)}$ are given by

$$ds^2 = h^{-1/2}(x, y, r) q_{\mu\nu}(X) dx^\mu dx^\nu + h^{1/2}(x, y, r) [\gamma_{ij}(Y) dy^i dy^j + h_K(r) u_{ab}(Z) dz^a dz^b + h_K^{-1}(r) (dv + A_a dz^a)^2], \quad (2a)$$

$$u_{ab}(Z) dz^a dz^b = dr^2 + r^2 w_{mn}(Z') dp^m dp^n, \quad (2b)$$

$$F_{(5)} = (1 \pm *) d(h^{-1}) \wedge \Omega(X), \quad (2c)$$

where $q_{\mu\nu}$ is a four-dimensional metric which depends only on the four-dimensional coordinates x^μ , γ_{ij} is a two-dimensional metric which depends only on the two-dimensional coordinates y^i , u_{ab} is a three-dimensional metric which depends only on the three-dimensional coordinates z^a , w_{mn} is a two-dimensional metric which depends only on the two-dimensional coordinates p^m , and the volume 4-form $\Omega(X)$ is given by

$$\Omega(X) = \sqrt{-q} dt \wedge dx^1 \wedge dx^2 \wedge dx^3. \quad (3)$$

Using the assumptions (2), the Einstein equations are given by

$$R_{\mu\nu}(X) - h^{-1}D_\mu D_\nu h + \frac{1}{4}q_{\mu\nu}h^{-2} \left[h\Delta_X h + \Delta_Y h + h_K^{-1} \left(\partial_r^2 h + \frac{2}{r}\partial_r h \right) \right] = 0, \quad (4a)$$

$$h^{-1}\partial_\mu \partial_i h = 0, \quad (4b)$$

$$h^{-1}\partial_\mu \partial_r h = 0, \quad (4c)$$

$$R_{ij}(Y) - \frac{1}{4}\gamma_{ij}h^{-1} \left[h\Delta_X h + \Delta_Y h + h_K^{-1} \left(\partial_r^2 h + \frac{2}{r}\partial_r h \right) \right] = 0, \quad (4d)$$

$$h_K \Delta_X h + h_K h^{-1} \left[\Delta_Y h + h_K^{-1} \left(\partial_r^2 h + \frac{2}{r}\partial_r h \right) \right] + 2h_K^{-1} \left(\partial_r^2 h_K + \frac{2}{r}\partial_r h_K \right) = 0, \quad (4e)$$

$$\begin{aligned} R_{mn}(Z') - w_{mn}(Z') - \frac{1}{4}h_K (r^2 w_{mn} + h_K^{-2} A_m A_n) [\Delta_X h \\ + h^{-1} \left\{ \Delta_Y h + h_K^{-1} \left(\partial_r^2 h + \frac{2}{r}\partial_r h \right) \right\} + 2h_K^{-1} \left(\partial_r^2 h + \frac{2}{r}\partial_r h \right)] \\ - \frac{1}{4}h_K^{-1} (r^2 w_{mn} - h_K^{-2} A_m A_n) \left(\partial_r^2 h_K + \frac{2}{r}\partial_r h_K \right) = 0, \end{aligned} \quad (4f)$$

$$h_K^{-1} \left[\Delta_X h + h^{-1} \left\{ \Delta_Y h + h_K^{-1} \left(\partial_r^2 h + \frac{2}{r}\partial_r h \right) \right\} - 2h_K^{-2} \left(\partial_r^2 h_K + \frac{2}{r}\partial_r h_K \right) \right] = 0, \quad (4g)$$

where we used the condition $dh_K = *_Z dA_{(1)}$, and D_μ is the covariant derivative with respect to the metric $q_{\mu\nu}(X)$, Δ_Y is the Laplace operators on the space of Y , and $R_{\mu\nu}(X)$, $R_{ij}(Y)$ and $R_{mn}(Z')$ are the Ricci tensors for the metrics $q_{\mu\nu}(X)$, $\gamma_{ij}(Y)$ and $w_{mn}(Z')$, respectively. From Eqs. (4b), (4c), we see that the function h becomes

$$h(x, y, r) = h_0(x) + h_1(y, r). \quad (5)$$

With this form of h , the other components of the Einstein Eqs. (4) are rewritten as

$$R_{\mu\nu}(X) - h^{-1}D_\mu D_\nu h_0 + \frac{1}{4}q_{\mu\nu}h^{-2} \left[h\Delta_X h_0 + \Delta_Y h_1 + h_K^{-1} \left(\partial_r^2 h_1 + \frac{2}{r}\partial_r h_1 \right) \right] = 0, \quad (6a)$$

$$R_{ij}(Y) - \frac{1}{4}\gamma_{ij}h^{-1} \left[h\Delta_X h_0 + \Delta_Y h_1 + h_K^{-1} \left(\partial_r^2 h_1 + \frac{2}{r}\partial_r h_1 \right) \right] = 0, \quad (6b)$$

$$h_K \Delta_X h_0 + h_K h^{-1} \left[\Delta_Y h_1 + h_K^{-1} \left(\partial_r^2 h_1 + \frac{2}{r}\partial_r h_1 \right) \right] + 2h_K^{-1} \left(\partial_r^2 h_K + \frac{2}{r}\partial_r h_K \right) = 0, \quad (6c)$$

$$\begin{aligned} R_{mn}(Z') - w_{mn}(Z') - \frac{1}{4} (h_K r^2 w_{mn} + h_K^{-1} A_m A_n) [\Delta_X h_0 \\ + h^{-1} \left\{ \Delta_Y h_1 + h_K^{-1} \left(\partial_r^2 h_1 + \frac{2}{r}\partial_r h_1 \right) \right\} + 2h_K^{-1} \left(\partial_r^2 h_K + \frac{2}{r}\partial_r h_K \right)] = 0, \end{aligned} \quad (6d)$$

$$h_K^{-1} \left[\Delta_X h_0 + h^{-1} \left\{ \Delta_Y h_1 + h_K^{-1} \left(\partial_r^2 h_1 + \frac{2}{r}\partial_r h_1 \right) \right\} - 2h_K^{-1} \left(\partial_r^2 h_K + \frac{2}{r}\partial_r h_K \right) \right] = 0. \quad (6e)$$

Now we consider the gauge field equations. Under the assumption (2), the equation of

motion for the gauge field (1b) gives

$$\left[h_K \Delta_Y h_1 + \left(\partial_r^2 h_1 + \frac{2}{r} \partial_r h_1 \right) \right] \Omega(Y) \wedge dr \wedge \Omega(Z') \wedge dv = 0, \quad (7)$$

where we have used (5), and the volume 2-forms $\Omega(Y)$, $\Omega(Z')$ are defined by

$$\Omega(Y) = \sqrt{\gamma} dy^1 \wedge dy^2, \quad (8a)$$

$$\Omega(Z') = \sqrt{w} dp^1 \wedge dp^2. \quad (8b)$$

Hence, the gauge field equation gives

$$\Delta_Y h_1 + h_K^{-1} \left(\partial_r^2 h_1 + \frac{2}{r} \partial_r h_1 \right) = 0. \quad (9)$$

Let us go back to the Einstein Eqs. (6). If $F_{(5)} = 0$, the function h_1 becomes trivial, and the internal space is no longer warped [6]. On the other hand, for $F_{(5)} \neq 0$, the first term in Eq. (6a) depends only on x , whereas the rest on x as well as y and r . Thus Eqs. (6) together with (9) give

$$R_{\mu\nu}(X) = 0, \quad R_{ij}(Y) = 0, \quad R_{mn}(Z') = w_{mn}(Z'), \quad (10a)$$

$$h(x, z) = h_0(x) + h_1(y, r); \quad D_\mu D_\nu h_0 = 0, \quad \Delta_Y h_1 + h_K^{-1} \left(\partial_r^2 h_1 + \frac{2}{r} \partial_r h_1 \right) = 0, \quad (10b)$$

$$\partial_r^2 h_K + \frac{2}{r} \partial_r h_K = 0. \quad (10c)$$

As a special example, we consider the case

$$q_{\mu\nu} = \eta_{\mu\nu}, \quad \gamma_{ij} = \delta_{ij}, \quad w_{mn}(Z') dp^m dp^n = d\Omega_{(2)}^2, \quad (11)$$

where $\eta_{\mu\nu}$ is the four-dimensional Minkowski metric, and δ_{ij} is the two-dimensional Euclidean metric, and $d\Omega_{(2)}^2$ is the metric of two-dimensional sphere. In this case, the solution for h can be obtained explicitly as

$$h(x, y, r) = c_\mu x^\mu + \bar{c} + \sum_\ell \frac{M_\ell}{(|y^i - y_\ell^i|^2 + 4M_K r)^2}, \quad (12a)$$

$$h_K(r) = \frac{M_K}{r}, \quad (12b)$$

where c_μ , \bar{c} , M_ℓ and M_K are constant parameters, and the constant y_ℓ^i denotes the position of the brane.

If the D3-brane is located at the origin of Y space, we have

$$h(x, y, r) = c_\mu x^\mu + \bar{c} + \frac{M}{(y^2 + 4M_K r)^2}, \quad (13)$$

where $y^2 = \delta_{ij}y^i y^j$. If we use the following coordinate transformation

$$y^1 = \zeta \cos \psi \cos \theta, \quad y^2 = \zeta \cos \psi \sin \theta, \quad r = \frac{1}{4} M_K^{-1} \zeta^2 \sin^2 \psi, \quad (14)$$

the ten-dimensional metric becomes

$$ds^2 = h^{-1/2}(x, \zeta) q_{\mu\nu}(X) dx^\mu dx^\nu + h^{1/2}(x, \zeta) \left[d\zeta^2 + \zeta^2 \{ d\psi^2 + \cos^2 \psi d\theta^2 + \frac{1}{4} \sin^2 \theta (d\Omega_{(2)}^2 + M_K^{-2} (dv + A_a dz^a)^2) \} \right], \quad (15)$$

where h is given by

$$h(x, \zeta) = c_\mu x^\mu + \bar{c} + \frac{M}{\zeta^4}. \quad (16)$$

Then the metric (15) in the limit $\zeta \rightarrow 0$ gives

$$ds^2 = M^{-1/2} \zeta^2 q_{\mu\nu}(X) dx^\mu dx^\nu + M^{1/2} \frac{d\zeta^2}{\zeta^2} + M^{1/2} \left[d\psi^2 + \cos^2 \psi d\theta^2 + \frac{1}{4} \sin^2 \theta \{ d\Omega_{(2)}^2 + M_K^{-2} (dv + A_a dz^a)^2 \} \right]. \quad (17)$$

The solution (17) in the type IIB supergravity theory enters in this particular string theory in much the same way that static spacetime physics (with general relativity as the near horizon limit) arises in conventional string theory.

B. Dynamical D3-wave solution

We present the time dependent D3-brane and wave solution. The field equations are given by

$$R_{MN} = \frac{1}{4 \cdot 4!} F_{MA_2 \dots A_5} F_N^{A_2 \dots A_5}, \quad (18a)$$

$$dF_{(5)} = 0, \quad F_{(5)} = *F_{(5)}, \quad (18b)$$

where $*$ is the Hodge operator in the ten-dimensional spacetime. We assume that the ten-dimensional metric takes the form

$$ds^2 = h^{1/2}(z) \left[-h^{-1}(z) h_w^{-1}(t, y, z) dt^2 + h^{-1}(z) h_w(t, y, z) \{ (h_w^{-1}(t, y, z) - 1) dt + dx \}^2 + h^{-1}(z) \gamma_{ij}(Y) dy^i dy^j + u_{ab}(Z) dz^a dz^b \right], \quad (19)$$

where γ_{ij} is a two-dimensional metric which depends only on the two-dimensional coordinates y^i , and u_{ab} is a six-dimensional metric which depends only on the six-dimensional coordinates z^a .

We also assume that the gauge field strength $F_{(5)}$ is given by

$$F_{(5)} = (1 \pm *)d[h^{-1}(t, y, z) \wedge dt \wedge dx \wedge \Omega(Y)], \quad (20)$$

where $\Omega(Y)$ denotes the volume 2-form

$$\Omega(Y) = \sqrt{\gamma} dy^1 \wedge dy^2. \quad (21)$$

Here, γ is the determinant of the metric γ_{ij} .

TABLE II: Dynamical D3-brane and pp-wave system in the metric (19). Here \circ denotes the worldvolume coordinate and \star denotes the wave coordinate respectively.

	0	1	2	3	4	5	6	7	8	9
D3	\circ	\circ	\circ	\circ						
W	\circ	\star								
x^N	t	x	y^1	y^2	z^1	z^2	z^3	z^4	z^5	z^6

Let us first consider the Einstein Eqs. (18a). In terms of the ansatz for fields (19) and (20), the Einstein equations are written by

$$\begin{aligned} & -\frac{1}{2}(h_w + 6)h^{-1}\partial_t^2 h + (2 - h_w)\partial_t^2 h_w + \Delta_Y h_w + h^{-1}\Delta_Z h_w + 2(2 - h_w)h^{-2}\Delta_Z h \\ & - \frac{1}{2}h_w(h_w - 2)\partial_t \ln h_w \partial_t \ln h - (h_w - 2)\partial_t \ln h \partial_t h_w = 0, \end{aligned} \quad (22a)$$

$$\partial_t \partial_i h_w = 0, \quad (22b)$$

$$\partial_t \partial_a h_w + h^{-1}\partial_t \partial_a h = 0, \quad (22c)$$

$$\begin{aligned} & -\frac{1}{2}h_w^2 h^{-1}\partial_t^2 h + h_w \partial_t^2 h_w - \Delta_Y h_w - h^{-1}\Delta_Z h_w - \frac{1}{2}h_w h^{-2}\Delta_Z h \\ & - h_w^2 \partial_t \ln h_w \partial_t \ln h = 0, \end{aligned} \quad (22d)$$

$$R_{ij}(Y) - \frac{1}{4}h^{-1}h_w \gamma_{ij} (\partial_t^2 h + \partial_t \ln h_w \partial_t \ln h) + \frac{1}{4}h^{-2}\gamma_{ij}\Delta_Z h = 0, \quad (22e)$$

$$R_{ab}(Z) - \frac{1}{4}h^{4/N}h_w u_{ab} (h^{-1}\partial_t^2 h + \partial_t \ln h_w \partial_t \ln h) - \frac{1}{2}h^{-1}\Delta_Z h = 0, \quad (22f)$$

where Δ_Y , Δ_Z are the Laplace operators on Y , Z space, and $R_{ij}(Y)$ and $R_{ab}(Z)$ are the Ricci tensors constructed from the metrics $\gamma_{ij}(Y)$, $u_{ab}(Z)$, respectively. From Eqs. (22b),

and (22c), h can be expressed as

$$h = h_0(t) + h_1(z), \quad h_w = h_w(y, z), \quad \text{For } \partial_t h_w = 0, \quad (23a)$$

$$h = h(z), \quad h_w = k_0(t) + k_1(y, z), \quad \text{For } \partial_t h = 0. \quad (23b)$$

Next we consider the gauge field Eqs. (18b). Under the assumptions (20), we find

$$d[\partial_a h (*_Z dz^a)] = 0, \quad (24)$$

where $*_Z$ denotes the Hodge operator on Z , and $\Omega(Z)$ denotes the volume 6-form :

$$\Omega(Z) = \sqrt{u} dz^1 \wedge dz^2 \wedge \cdots \wedge dz^6. \quad (25)$$

Then, the Eq. (24) gives

$$\Delta_Z h = 0, \quad \partial_t \partial_a h = 0, \quad (26)$$

where Δ_Z is the Laplace operators on the space of Z . For the case $\partial_t h_w = 0$, the Einstein Eqs. (22) are thus rewritten as

$$-\frac{1}{2}(h_w + 6)h^{-1}\partial_t^2 h_0 + \Delta_Y h_w + h^{-1}\Delta_Z h_w + 2(2 - h_w)h^{-2}\Delta_Z h_1 = 0, \quad (27a)$$

$$\frac{1}{2}h_w^2 h^{-1}\partial_t^2 h_0 + \Delta_Y h_w + h^{-1}\Delta_Z h_w + \frac{1}{2}h_w h^{-2}\Delta_Z h_1 = 0, \quad (27b)$$

$$R_{ij}(Y) - \frac{1}{4}h^{-1}h_w \gamma_{ij} \partial_t^2 h_0 + \frac{1}{4}h^{-2}\Delta_Z h_1 = 0, \quad (27c)$$

$$R_{ab}(Z) - \frac{1}{4}u_{ab}h_w \partial_t^2 h_0 - \frac{1}{4}h^{-1}\Delta_Z h_1 = 0. \quad (27d)$$

Hence, the field equations reduce to

$$R_{ij}(Y) = 0, \quad R_{ab}(Z) = 0, \quad (28a)$$

$$h = h_0(t) + h_1(z), \quad (28b)$$

$$\partial_t^2 h_0 = 0, \quad \partial_t h_0 \partial_t h_w = 0, \quad \Delta_Z h_1 = 0, \quad (28c)$$

$$h\Delta_Y h_w + \Delta_Z h_w = 0. \quad (28d)$$

We can also choose the solution in which the function h_w depends on t . Then, we have

$$R_{ij}(Y) = 0, \quad R_{ab}(Z) = 0, \quad (29a)$$

$$h = h(z), \quad h_w = k_0(t) + k_1(y, z), \quad (29b)$$

$$\partial_t^2 k_0 = 0, \quad \Delta_Y k_1 + \Delta_Z k_1 = 0. \quad (29c)$$

If $F_{(5)} = 0$, the function h_1 becomes trivial.

Here we focus on the case

$$\gamma_{ij} = \delta_{ij}, \quad u_{ab} = \delta_{ab}, \quad h = h(z), \quad (30)$$

where δ_{ij} , δ_{ab} are the two-, six-dimensional Euclidean metrics, respectively. Then, the solution for h and h_w can be written by

$$h_w(t, z) = ct + \bar{c} + \sum_{\ell} \frac{M_{\ell}}{[|y^i - y_{\ell}^i|^2 + M|z^a - z_0^a|^{-2}]^{-1}}, \quad (31a)$$

$$h(z) = \frac{M}{|z^a - z_0^a|^4}, \quad (31b)$$

where c , \bar{c} , M_{ℓ} and M are constant parameters, and the constant y_{ℓ}^i and z_0^a denote the positions of the branes. If the D3-brane is located at the origin of Y and Z spaces, the solution is given by

$$h_w(t, z) = ct + \bar{c} + \frac{M_w}{(|y^i|^2 + M|z^a|^{-2})^{-1}}, \quad (32a)$$

$$h(z) = \frac{M}{|z^a|^4}. \quad (32b)$$

Now we introduce coordinates

$$y^1 = \frac{1}{r} \cos \theta \cos \alpha, \quad y^2 = \frac{1}{r} \sin \theta \cos \alpha, \quad z^a = \frac{r M^{1/2}}{\sin \alpha} \mu^a, \quad (33)$$

where μ^a is defined as

$$\mu_a \mu^a = 1, \quad d\Omega_{(5)}^2 = d\mu_a d\mu^a. \quad (34)$$

Here $d\Omega_{(5)}^2$ is the line element of the unit 5-sphere. In terms of (32) the metric of the D3-wave system becomes

$$ds^2 = M^{1/2} \sin^{-2} \alpha [ds_{\text{AdS}_3}^2 + d\alpha^2 + \cos^2 \alpha d\theta^2 + \sin^2 \alpha d\Omega_{(5)}^2], \quad (35a)$$

$$ds_{\text{AdS}}^2 = -r^2 h_w^{-1} dt^2 + r^2 h_w [(h_w^{-1} - 1) dt + dx]^2 + r^{-2} dr^2, \quad (35b)$$

$$h_w = ct + \bar{c} + \frac{M_w}{r^2}. \quad (35c)$$

Since the metric (35) is the extremal BTZ black hole, which is locally AdS_3 , the dynamical D3-wave system is a warped product of AdS_3 with a 7-sphere.

If we apply T duality in the x direction of the ten-dimensional spacetime, the D3-brane and pp wave become D2-brane, F1 string, respectively. Then we can obtain the solution for dynamical F1-D2 brane followed by T-duality map in the time dependent D3-wave system. We will show how to obtain the dynamical F1-D2 brane solution via the T-duality in the next subsection.

C. Dynamical F1-D2 brane solution

Now we consider the dynamical F1-D2 brane solution in ten-dimensional IIA theory. We assume that the ten-dimensional metric takes the form

$$ds^2 = h_F^{1/4}(t, y, z) h^{3/8}(z) \left[-h_F^{-1}(t, y, z) h^{-1}(z) dt^2 + h_F^{-1}(t, y, z) dx^2 + h^{-1}(z) \gamma_{ij}(Y) dy^i dy^j + u_{ab}(Z) dz^a dz^b \right], \quad (36)$$

where γ_{ij} is a two-dimensional metric which depends only on the two-dimensional coordinates y^i , and finally u_{ab} is a six-dimensional metric which depends only on the six-dimensional coordinates z^a .

We also assume that the scalar and gauge field strengths $H_{(3)}, F_{(4)}$ are given by

$$e^\phi = h_F^{-1/2} h^{1/4}, \quad (37a)$$

$$H_{(3)} = d(h_F^{-1}) \wedge dt \wedge dx, \quad (37b)$$

$$F_{(4)} = d(h^{-1}) \wedge dt \wedge \Omega(Y), \quad (37c)$$

where $\Omega(Y)$ denotes the volume 2-form

$$\Omega(Y) = \sqrt{\gamma} dy^1 \wedge dy^2. \quad (38)$$

Here, γ is the determinant of the metric γ_{ij} .

TABLE III: Dynamical F1-D2 brane system in the metric (36). Here \circ denotes the worldvolume coordinate.

	0	1	2	3	4	5	6	7	8	9
F1	\circ			\circ						
D2	\circ	\circ	\circ							
x^N	t	y^1	y^2	x	z^1	z^2	z^3	z^4	z^5	z^6

By using the ansatz for fields (36) and (37), we get

$$R_{ij}(Y) = 0, \quad R_{ab}(Z) = 0, \quad (39a)$$

$$h_F = h_0(t) + h_1(y, z), \quad \partial_t^2 h_0 = 0, \quad h \Delta_Y h_1 + \Delta_Z h_1 = 0, \quad \Delta_Z h = 0, \quad (39b)$$

where Δ_Y, Δ_Z are the Laplace operators on Y, Z space, and $R_{ij}(Y)$ and $R_{ab}(Z)$ are the Ricci tensors constructed from the metrics $\gamma_{ij}(Y), u_{ab}(Z)$, respectively. Let us consider the case

$$\gamma_{ij} = \delta_{ij}, \quad u_{ab} = \delta_{ab}, \quad (40)$$

where δ_{ij} , δ_{ab} are the two-, six-dimensional Euclidean metrics, respectively. The solution of h and h_F can be expressed as

$$h_F(t, y, z) = \bar{c}t + \tilde{c} + \sum_{\ell} M_{\ell} [|y^i - y_{\ell}^i|^2 + M|z^a - z_0^a|^{-2}] , \quad (41a)$$

$$h(z) = \frac{M}{|z^a - z_0^a|^4} , \quad (41b)$$

where \bar{c} , \tilde{c} , M_{ℓ} and M are constant parameters, and the constant y_{ℓ}^i and z_0^a denote the positions of the branes. If we consider the case where F1-brane is located at the origin of the Y, Z spaces and use a coordinate transformation (33), we have

$$h_F(t, r) = \bar{c}t + \tilde{c} + \frac{M_F}{r^2} , \quad (42a)$$

$$h(r) = \frac{\sin^4 \alpha}{Mr^4} , \quad (42b)$$

where M_F is constant. In the near horizon limit $r \rightarrow 0$, the metric becomes

$$ds^2 = M_F^{1/4} M^{5/8} (\sin \alpha)^{-5/2} \left[-\frac{r^4}{M_F} dt^2 + \frac{dr^2}{r^2} + d\alpha^2 + \cos^2 \alpha d\theta^2 \right. \quad (43)$$

$$\left. + \sin^2 \alpha d\Omega_{(5)}^2 + (M_F M)^{-1} \sin^4 \alpha (dy^1)^2 \right] . \quad (44)$$

Then, the ten-dimensional metric is a warped product of the AdS_2 with an eight-dimensional internal space.

We can construct the solution of the F1-D2 brane system (36) in terms of T-duality map in the D3-pp wave solution. We start from the dynamical D3-pp wave solution in the string frame in the type IIB theory

$$ds_{(B)}^2 = h^{-1/2}(z) [-dt^2 + dx^2 + \{h_W(t, y, z) - 1\} (dt - dx)^2 \\ + \delta_{ij}(Y) dy^i dy^j + h(z) \delta_{ab}(Z) dz^a dz^b] , \quad (45)$$

$$C_{(4)} = h^{-1}(z) dt \wedge dx \wedge dy^1 \wedge dy^2 + \omega_{(4)} , \quad (46)$$

where the warp factor h and h_W are given by

$$h_W(t, y, z) = At + B + \frac{M_W}{(|y^i - y_{\ell}^i|^2 + M|z^a - z_0^a|^{-2})^{-1}} , \quad (47a)$$

$$h(z) = \frac{M}{|z^a - z_0^a|^4} . \quad (47b)$$

Here, the constant y_{ℓ}^i and z_0^a denote the positions of the branes.

The 4-form $\omega_{(4)}$ satisfies

$$d\omega_{(4)} = \pm \partial_a h *_{\mathbb{Z}} (dz^a) . \quad (48)$$

Here $*_{\mathbb{Z}}$ denotes the Hodge operator on \mathbb{Z} . Now we will obtain the dynamical solution of a D3-pp after we apply T duality in the x direction of the ten-dimensional spacetime. The ten-dimensional T duality map from the type IIB theory to type IIA theory is given by [26–29]

$$\begin{aligned} g_{xx}^{(A)} &= \frac{1}{g_{xx}^{(B)}}, & g_{\mu\nu}^{(A)} &= g_{\mu\nu}^{(B)} - \frac{g_{x\mu}^{(B)} g_{x\nu}^{(B)} - B_{x\mu}^{(B)} B_{x\nu}^{(B)}}{g_{xx}^{(B)}}, & g_{x\mu}^{(A)} &= -\frac{B_{x\mu}^{(B)}}{g_{xx}^{(B)}}, \\ e^{2\phi_{(A)}} &= \frac{e^{2\phi_{(B)}}}{g_{xx}^{(B)}}, & C_{\mu} &= C_{x\mu} + C_{(0)} B_{x\mu}^{(B)}, & C_x &= -C_{(0)}, \\ B_{\mu\nu}^{(A)} &= B_{\mu\nu}^{(B)} + 2 \frac{B_{x[\mu} g_{\nu]x}^{(B)}}{g_{xx}^{(B)}}, & B_{x\mu}^{(A)} &= -\frac{g_{x\mu}^{(B)}}{g_{xx}^{(B)}}, & C_{x\mu\nu} &= C_{\mu\nu} + 2 \frac{C_{x[\mu} g_{\nu]x}^{(B)}}{g_{xx}^{(B)}}, \\ C_{\mu\nu\rho} &= C_{\mu\nu\rho x} + \frac{3}{2} \left(C_{x[\mu} B_{\nu\rho]}^{(B)} - B_{x[\mu} C_{\nu\rho]}^{(B)} - 4 \frac{B_{x[\mu} C_{|x|\nu} g_{\rho x}^{(B)}}{g_{xx}^{(B)}} \right), \end{aligned} \quad (49)$$

where x is the coordinate to which the T dualization is applied in the D3-pp wave solution, and μ , ν , and ρ denote the coordinates other than x . In terms of the T-duality map (49), the D3-pp wave solution (19) becomes

$$ds_{(A)}^2 = h_F^{1/4} h_2^{3/8} \left[-(h_F h_2)^{-1} dt^2 + h_F^{-1} dx^2 + h_2^{-1} \delta_{ij}(Y) dy^i dy^j + \delta_{ab}(Z) dz^a dz^b \right], \quad (50a)$$

$$e^{\phi_{(A)}} = h_F^{-1/2} h_2^{1/4}, \quad (50b)$$

$$B_{(2)} = h_F^{-1} dt \wedge dx, \quad (50c)$$

$$C_{(3)} = h_2^{-1} dt \wedge dy^1 \wedge dy^2, \quad (50d)$$

where $ds_{(A)}^2$ is the ten-dimensional metric in the Einstein frame, and h and h_F can be written by

$$h_F(t, y, z) = h_W(t, y, z), \quad (51a)$$

$$h_2(z) = h(z). \quad (51b)$$

Finally we obtain the solution (36) and (37) which are derived from the dynamical D3-pp wave solution via T-duality.

III. ADS SPACETIME FROM M2-BRANE SOLUTIONS

In this section, we apply the above solutions to M2-brane in the eleven-dimensional theory. We also discuss the dynamical the D-brane solutions in ten dimensions in terms of dimensional reduction of the eleven-dimensional theory.

A. Dynamical M2-brane and KK monopole solution

The eleven-dimensional action which contains the metric g_{MN} , and 4-form field strength $F_{(4)}$ is given by

$$S = \frac{1}{2\kappa^2} \int \left[R * \mathbf{1} - \frac{1}{2 \cdot 4!} F_{(4)} \wedge * F_{(4)} \right], \quad (52)$$

where κ^2 is the eleven-dimensional gravitational constant, $*$ is the Hodge operator in the eleven-dimensional space-time. The field strengths $F_{(4)}$ is given by the 3-form gauge potential

$$F_{(4)} = dC_{(3)}. \quad (53)$$

The field equations are given by

$$R_{MN} = \frac{1}{2 \cdot 4!} \left[4F_{MABC} F_N{}^{ABC} - \frac{1}{2} g_{MN} F_{(4)}^2 \right], \quad (54a)$$

$$d[*F_{(4)}] = 0, \quad dF_{(4)} = 0. \quad (54b)$$

We present the solution involving the M2-brane and KK monopole system. We take the eleven-dimensional metric to be [30]

$$ds^2 = h_2^{-2/3}(x, y, z) q_{\mu\nu}(X) dx^\mu dx^\nu + h_2^{1/3}(x, y, z) [\gamma_{ij}(Y) dy^i dy^j + h_K(z) u_{ab}(Z) dz^a dz^b + h_K^{-1}(z) (dv + A_a dz^a)^2], \quad (55a)$$

$$u_{ab}(Z) dz^a dz^b = dr^2 + r^2 w_{mn}(Z') dp^m dp^n, \quad (55b)$$

where $q_{\mu\nu}$ is a three-dimensional metric which depends only on the three-dimensional coordinates x^μ , γ_{ij} is a four-dimensional metric which depends only on the four-dimensional coordinates y^i , and u_{ab} is a three-dimensional metric which depends only on the three-dimensional coordinates z^a , and finally w_{mn} is a two-dimensional metric which depends only on the two-dimensional coordinates p^m .

The 4-form gauge field strength $F_{(4)}$ is assumed to be

$$F_{(4)} = d[h_2^{-1}(x, y, z)] \wedge \Omega(X), \quad (56)$$

where the volume 3-form $\Omega(X)$ is given by

$$\Omega(X) = \sqrt{-q} dx^0 \wedge dx^1 \wedge dx^2. \quad (57)$$

Here, q is the determinant of the metric $q_{\mu\nu}$.

TABLE IV: Dynamical M2-brane and KK monopole in the metric (55a). Here \circ denotes the worldvolume coordinate and \bullet denotes the fibre coordinate of the KK-monopole respectively.

	0	1	2	3	4	5	6	7	8	9	10
M2	\circ	\circ	\circ								
KK	\circ	\circ	\circ	\circ	\circ	\circ	\circ	\bullet			
x^N	t	x^1	x^2	y^1	y^2	y^3	y^4	v	z^1	z^2	z^3

By using the ansatz for fields (55), (56), and the condition $dh_K = *_Z dA_{(1)}$, the field equations give

$$R_{\mu\nu}(X) = 0, \quad R_{ij}(Y) = 0, \quad R_{ab}(Z) = 0, \quad R_{mn}(Z') = w_{mn}(Z'), \quad (58a)$$

$$h_2(x, y, z) = h_0(x) + h_1(y, z), \quad (58b)$$

$$D_\mu D_\nu h_0 = 0, \quad h_K \Delta_Y h_1 + \Delta_Z h_1 = 0, \quad \Delta_Z h_K = 0, \quad (58c)$$

where D_μ is the covariant derivative with respect to the metric $q_{\mu\nu}$, and Δ_Y , Δ_Z are the Laplace operators on Y , Z space, and $R_{\mu\nu}(X)$, $R_{ij}(Y)$ and $R_{ab}(Z)$ are the Ricci tensors with respect to the metrics $q_{\mu\nu}(X)$, $\gamma_{ij}(Y)$, $u_{ab}(Z)$, respectively.

Now we consider the case

$$q_{\mu\nu} = \eta_{\mu\nu}, \quad \gamma_{ij} = \delta_{ij}, \quad u_{ab} = \delta_{ab}, \quad (59)$$

where $\eta_{\mu\nu}$ is the three-dimensional Minkowski metric and δ_{ij} , δ_{ab} are the four-, three-dimensional Euclidean metrics, respectively. The solution for h_2 and h_K can be obtained explicitly as

$$h_2(x, y, r) = c_\mu x^\mu + \tilde{c} + \frac{M_2}{(|y^i - y_\ell^i|^2 + 4M_K r)^3}, \quad (60a)$$

$$h_K(r) = \frac{M_K}{r}, \quad (60b)$$

where c_μ , \tilde{c} , M_2 and M_K are constant parameters, and the constant y_ℓ^i denotes the position of the brane. Let us consider a coordinate transformation

$$y^i = \zeta \xi^i \cos \alpha, \quad r = \frac{1}{4} M_K^{-1} \zeta^2 \sin^2 \alpha, \quad (61)$$

where $\xi_i \xi^i = 1$. In terms of (61), the eleven-dimensional metric (55) becomes

$$ds^2 = h_2^{-2/3} \eta_{\mu\nu}(X) dx^\mu dx^\nu + h_2^{1/3} [d\zeta^2 + \zeta^2 ds^2(M_7)], \quad (62)$$

where h_2 and $ds^2(M_7)$ are given by

$$h_2 = c_\mu x^\mu + \tilde{c} + \frac{M_2}{\zeta^6}, \quad h_K(r) = \frac{4M_K^2}{\zeta^2 \sin^2 \alpha}, \quad (63)$$

$$ds^2(M_7) = d\alpha^2 + \cos^2 \alpha d\Omega_{(3)}^2 + \frac{1}{4} \sin^2 \alpha [d\Omega_{(2)}^2 + M_K^{-2} (dv + A_a dz^a)^2]. \quad (64)$$

Here $d\Omega_{(2)}^2$ and $d\Omega_{(3)}^2 = d\xi_i d\xi^i$ are the line element of the unit 2- and 3-sphere, respectively. In the near horizon limit $\zeta \rightarrow 0$, the metric becomes $\text{AdS}_4 \times M_7$. After performing Kaluza-Klein reduction on the v coordinate, the solution (60) gives a dynamical D2-D6 brane. We will discuss the dynamical D2-D6 brane solution in the next subsection.

B. Dynamical D2-D6 brane solution

Now we discuss the dynamical D2-D6 brane solutions in ten-dimensional type IIA string theory. The action for IIA theory in the Einstein frame can be written as

$$S = \frac{1}{2\kappa^2} \int \left(R * 1 - \frac{1}{2} d\phi \wedge * d\phi - \frac{1}{2 \cdot 2!} e^{3\phi/2} F_{(2)} \wedge * F_{(2)} - \frac{1}{2 \cdot 4!} e^{\phi/2} F_{(4)} \wedge * F_{(4)} \right), \quad (65)$$

where κ^2 is the ten-dimensional gravitational constant, $*$ is the Hodge dual operator in the ten-dimensional spacetime, and $F_{(2)}$, $F_{(4)}$ are RR 2-form, RR 4-form field strengths, respectively. The expectation values of fermionic fields are assumed to be zero.

After variations with respect to the metric, the scalar field, and the gauge field, the field equations are given by

$$R_{MN} = \frac{1}{2} \partial_M \phi \partial_N \phi + \frac{1}{2 \cdot 2!} e^{3\phi/2} \left(2F_{MA} F_N{}^A - \frac{1}{8} g_{MN} F_{(2)}^2 \right) + \frac{1}{2 \cdot 4!} e^{\phi/2} \left(4F_{MABC} F_N{}^{ABC} - \frac{3}{8} g_{MN} F_{(4)}^2 \right), \quad (66a)$$

$$d * d\phi = \frac{3}{4 \cdot 2!} e^{3\phi/2} F_{(2)} \wedge * F_{(2)} + \frac{1}{4 \cdot 4!} e^{\phi/2} F_{(4)} \wedge * F_{(4)}, \quad (66b)$$

$$d(e^{3\phi/2} * F_{(2)}) = 0, \quad (66c)$$

$$d(e^{\phi/2} * F_{(4)}) = 0. \quad (66d)$$

The ten-dimensional metric is assumed to be

$$ds^2 = h_2^{3/8}(x, y, z) h_6^{7/8}(z) [h_2^{-1}(x, y, z) h_6^{-1}(z) q_{\mu\nu}(X) dx^\mu dx^\nu + h_6^{-1}(z) \gamma_{ij}(Y) dy^i dy^j + u_{ab}(Z) dz^a dz^b], \quad (67)$$

where $q_{\mu\nu}$ is a three-dimensional metric which depends only on the three-dimensional coordinates y^i , γ_{ij} is a four-dimensional metric which depends only on the four-dimensional coordinates y^i , and u_{ab} is a three-dimensional metric which depends only on the three-dimensional coordinates z^a . The metric form Eq. (67) is a straightforward generalization of the case of a static D2-D6 brane system with a dilaton coupling [24]. Furthermore, we assume that the scalar field ϕ and the gauge field strengths are given by

$$e^\phi = h_2^{1/4} h_6^{-3/4}, \quad (68a)$$

$$F_{(2)} = e^{-3\phi/2} * [d(h_6^{-1}) \wedge \Omega(X) \wedge \Omega(Y)] , \quad (68b)$$

$$F_{(4)} = d(h_2^{-1}) \wedge \Omega(X) , \quad (68c)$$

where $\Omega(X)$ and $\Omega(Y)$ denote the volume 3- and 4-forms,

$$\Omega(X) = \sqrt{-q} dx^0 \wedge dx^1 \wedge dx^2, \quad (69)$$

$$\Omega(Y) = \sqrt{\gamma} dy^1 \wedge dy^2 \wedge dy^3 \wedge dy^4, \quad (70)$$

respectively.

TABLE V: Dynamical D2-D6 brane system in the metric (67). Here \circ denotes the worldvolume coordinate.

	0	1	2	3	4	5	6	7	8	9
D2	\circ	\circ	\circ							
D6	\circ	\circ	\circ	\circ	\circ	\circ	\circ			
x^N	t	x^1	x^2	y^1	y^2	y^3	y^4	z^1	z^2	z^3

The field equations reduce to

$$R_{\mu\nu}(X) = 0, \quad R_{ij}(Y) = 0, \quad R_{ab}(Z) = 0, \quad (71a)$$

$$h_2(x, y, z) = K(x) + L(y, z), \quad h_6 = h_6(z);$$

$$D_\mu D_\nu K = 0, \quad \Delta_Z L(y, z) + h_6 \Delta_Y L(y, z) = 0, \quad \Delta_Z h_6 = 0, \quad (71b)$$

where D_μ is the covariant derivative with respect to the metric $q_{\mu\nu}$, and Δ_Y , Δ_Z are the Laplace operators on Y, Z space, and $R_{\mu\nu}(X)$, $R_{ij}(Y)$ and $R_{ab}(Z)$ are the Ricci tensors constructed from the metrics $q_{\mu\nu}(X)$, $\gamma_{ij}(Y)$, $u_{ab}(Z)$, respectively.

Let us consider the case

$$q_{\mu\nu} = \eta_{\mu\nu}, \quad \gamma_{ij} = \delta_{ij}, \quad u_{ab} = \delta_{ab}, \quad (72)$$

where $\eta_{\mu\nu}$ is the three-dimensional Minkowski metric, and δ_{ij} , δ_{ab} are the four-dimensional, three-dimensional Euclidean metric. In this case, the solution for h_2 and h_6 can be obtained explicitly as

$$h_2(x, z) = c_\mu x^\mu + \tilde{c} + \sum_l \frac{M_\ell}{(|y^i - y_\ell^i|^2 + 4M_6|z^a - z_0^a|)^3}, \quad h_6(z) = \frac{M_6}{|z^a - z_0^a|}, \quad (73)$$

where c_μ , \tilde{c} , M_6 , M_ℓ , y_ℓ^i and z_0^a are constant parameters.

Now we investigate the geometrical properties of the D2-D6 brane metric (72). We consider the case where the D2-brane is located at the origin of the Y space and use the coordinate

$$\delta_{ab} dz^a dz^b = dr^2 + r^2 d\Omega_{(2)}^2, \quad (74)$$

where $d\Omega_{(2)}^2$ is the line element of two-dimensional unit 2-sphere. Then we have

$$h_2(x, r) = c_\mu x^\mu + \tilde{c} + \frac{M_2}{(y^2 + 4M_6 r)^3}, \quad h_6(r) = \frac{M_6}{r}, \quad (75)$$

where $y^2 = \delta_{ij} y^i y^j$, and M_2 is constant.

In terms of a coordinate transformation

$$y^i = \zeta \xi^i \cos \alpha, \quad r = \frac{1}{4} M_6^{-1} \zeta^2 \sin^2 \alpha, \quad (76)$$

the eleven-dimensional metric (67) becomes

$$ds^2 = \left(\frac{\zeta \sin \alpha}{2M_6} \right)^{1/4} h_2^{3/8}(x, \zeta) \left[h_2^{-1}(x, \zeta) \eta_{\mu\nu}(X) dx^\mu dx^\nu + h_2(x, \zeta) \left\{ d\zeta^2 + \zeta^2 \left(d\alpha^2 + \cos^2 \alpha d\Omega_{(3)}^2 + \frac{1}{4} \sin^2 \alpha d\Omega_{(2)}^2 \right) \right\} \right], \quad (77a)$$

$$h_2(x, \zeta) = A_\mu x^\mu + B + \frac{M_2}{\zeta^6}, \quad h_6(\zeta) = \frac{4M_6^2}{\zeta^2 \sin^2 \alpha}, \quad (77b)$$

where $\xi_i \xi^i = 1$, and $d\Omega_{(3)}^2$ is the line elements of 3-spheres. Then, the metric (77a) in the limit $\zeta \rightarrow 0$ reads

$$ds^2 = M_2^{3/8} \left(\frac{\sin \alpha}{2M_6} \right)^{1/4} \left[\left(\frac{\zeta^4}{M_2} \right) \eta_{\mu\nu}(X) dx^\mu dx^\nu + \frac{d\zeta^2}{\zeta^2} + d\alpha^2 + \cos^2 \alpha d\Omega_{(3)}^2 + \frac{\sin^2 \alpha}{4} d\Omega_{(2)}^2 \right]. \quad (78)$$

Hence the ten-dimensional metric for $\alpha = 0$ and $\gamma_{ij} = \delta_{ij}$ becomes a warped $\text{AdS}_4 \times \text{S}^6$ spacetime.

C. Dynamical D2-D6-KK monopole solution

If we introduce the KK-monopole in the D2-D6 brane system, it is possible to obtain the dynamical solutions for D2-D6-KK monopole. Following the same procedure as the case of the D3-KK monopole, we can generalize the solution found in the previous subsection for the D2-D6 brane system. We look for solutions whose spacetime metric has the form

$$ds^2 = h_2^{3/8}(x, y, z) h_6^{7/8}(z) \left[h_2^{-1}(x, y, z) h_6^{-1}(z) q_{\mu\nu}(X) dx^\mu dx^\nu + h_6^{-1}(z) \gamma_{ij}(Y) dy^i dy^j + h_K(z) u_{ab}(Z) dz^a dz^b + h_K^{-1}(z) (dv + A_a dz^a)^2 \right], \quad (79)$$

where $q_{\mu\nu}$ is a three-dimensional metric which depends only on the three-dimensional coordinates y^i , γ_{ij} is a three-dimensional metric which depends only on the three-dimensional coordinates y^i , and u_{ab} is a three-dimensional metric which depends only on the three-dimensional coordinates z^a . The metric form Eq. (79) is a straightforward generalization of the case of a static D2-D6-KK monopole system with a dilaton coupling [24]. Furthermore, we assume that the scalar field ϕ and the gauge field strengths are given by

$$e^\phi = h_2^{1/4} h_6^{-3/4}, \quad (80a)$$

$$F_{(2)} = e^{-3\phi/2} * [d(h_6^{-1}) \wedge \Omega(X) \wedge \Omega(Y)], \quad (80b)$$

$$F_{(4)} = d(h_2^{-1}) \wedge \Omega(X), \quad (80c)$$

where $\Omega(X)$ and $\Omega(Y)$ denote the volume 3-form

$$\Omega(X) = \sqrt{-q} dx^0 \wedge dx^1 \wedge dx^2, \quad (81a)$$

$$\Omega(Y) = \sqrt{\gamma} dy^1 \wedge dy^2 \wedge dy^3, \quad (81b)$$

respectively. The brane configuration is given in Table VI.

TABLE VI: Time dependent D2-D6-KK monopole system. Here \circ denotes the worldvolume coordinate and \bullet denotes the fibre coordinate of the KK-monopole respectively.

Case		0	1	2	3	4	5	6	7	8	9
D2-D6-KK	D2	\circ	\circ	\circ							
	D6	\circ	\circ	\circ	\circ	\circ	\circ	\circ			
	KK	\circ	\circ	\circ				\bullet	\circ	\circ	\circ
	x^N	t	x^1	x^2	y^1	y^2	y^3	v	z^1	z^2	z^3

Using the form of metric (79) and the fields (80), the field equations reduce to

$$R_{\mu\nu}(X) = 0, \quad R_{ij}(Y) = 0, \quad R_{ab}(Z) = 0, \quad *_Z dA_a = dh_K, \quad (82a)$$

$$h_2(x, y, z) = K(x) + L(y, z), \quad h_6 = h_6(z), \quad h_K = h_K(y); \quad (82b)$$

$$D_\mu D_\nu K = 0, \quad \Delta_Z L(y, z) + h_6 h_K \Delta_Y L(y, z) = 0, \quad \Delta_Z h_6 = 0, \quad \Delta_Y h_K = 0, \quad (82c)$$

where D_μ is the covariant derivative with respect to the metric $q_{\mu\nu}$, and Δ_Y, Δ_Z are the Laplace operators on Y, Z space, and $R_{\mu\nu}(X), R_{ij}(Y)$ and $R_{ab}(Z)$ are the Ricci tensors with respect to the metrics $q_{\mu\nu}(X), \gamma_{ij}(Y), u_{ab}(Z)$, respectively. Now we assume that the metric of the ten-dimensional spacetime is given by

$$q_{\mu\nu} = \eta_{\mu\nu}, \quad \gamma_{ij} = \delta_{ij}, \quad u_{ab} = \delta_{ab}, \quad (83)$$

where $\eta_{\mu\nu}$ is the three-dimensional Minkowski metric, and δ_{ij}, δ_{ab} are the three-dimensional, three-dimensional Euclidean metric. In this case, the solution can be obtained explicitly as

$$h_2(x, y, z) = c_\mu x^\mu + \tilde{c} + \sum_l \frac{M_l}{(4M_K |y^i - y_\ell^i|^2 + 4M |z^a - z_0^a|)^3}, \quad (84a)$$

$$h_6(z) = \frac{M}{|z^a - z_0^a|}, \quad h_K(y) = \frac{M_K}{|y^i - y_\ell^i|}, \quad (84b)$$

where $c_\mu, \tilde{c}, M, M_K, M_l, y_\ell^i$ and z_0^a are constant parameters.

If we perform the T-duality on the coordinate v , the solution (84) leads to the dynamical D3-D5-NS5 brane system. Now we show how to obtain the D3-D5-NS5 brane solution via the T-duality. We start from the dynamical D2-D6-KK monopole solution in the string frame in the type IIA theory;

$$ds_{(A)}^2 = h_2^{1/2}(x, y, z) h_6^{1/2}(z) [h_2^{-1}(x, y, z) h_6^{-1}(z) \eta_{\mu\nu}(X) dx^\mu dx^\nu + h_6^{-1}(z) \delta_{ij}(Y) dy^i dy^j + h_K(z) \delta_{ab}(Z) dz^a dz^b + h_K^{-1}(z) (dv + A_a dz^a)^2], \quad (85a)$$

$$C_{(3)} = \pm h_2^{-1} dt \wedge dx^1 \wedge dx^2, \quad (85b)$$

$$C_{(1)} = \omega_{(1)}, \quad (85c)$$

$$e^{2\phi_{(A)}} = h_2^{1/2} h_6^{-3/2}, \quad (85d)$$

where $C_{(3)}$ and $C_{(1)}$ are gauge potentials for D2- and D4-branes, and $\omega_{(1)}$ satisfies

$$d\omega_{(1)} = \partial_a h_6 *_Z dz^a. \quad (86)$$

Here $*_Z$ is the Hodge operator in the Z space. Now we will obtain the dynamical solution of a D3-D5 brane after we apply T duality in the y direction of the ten-dimensional spacetime (85a). The ten-dimensional T duality map from the type IIA theory to type IIB theory is given by [26, 27]

$$\begin{aligned}
g_{yy}^{(B)} &= \frac{1}{g_{yy}^{(A)}}, & g_{\mu\nu}^{(B)} &= g_{\mu\nu}^{(A)} - \frac{g_{y\mu}^{(A)} g_{y\nu}^{(A)} - B_{y\mu}^{(A)} B_{y\nu}^{(A)}}{g_{yy}^{(A)}}, & g_{y\mu}^{(B)} &= -\frac{B_{y\mu}^{(A)}}{g_{yy}^{(A)}}, \\
e^{2\phi^{(B)}} &= \frac{e^{2\phi^{(A)}}}{g_{yy}^{(A)}}, & B_{\mu\nu}^{(B)} &= B_{\mu\nu}^{(A)} + 2\frac{g_{y[\mu}^{(A)} B_{\nu]y}^{(A)}}{g_{yy}^{(A)}}, & B_{y\mu}^{(B)} &= -\frac{g_{y\mu}^{(A)}}{g_{yy}^{(A)}}, \\
C_{\mu\nu} &= C_{\mu\nu y} - 2C_{[\mu} B_{\nu]y}^{(A)} + 2\frac{g_{y[\mu}^{(A)} B_{\nu]y}^{(A)} C_y}{g_{yy}^{(A)}}, & C_{y\mu} &= C_\mu - \frac{C_y^{(A)} g_{y\mu}^{(A)}}{g_{yy}^{(A)}}, \\
C_{\mu\nu\rho y} &= C_{\mu\nu\rho} - \frac{3}{2} \left(C_{[\mu} B_{\nu]\rho}^{(A)} - \frac{g_{y[\mu}^{(A)} B_{\nu]\rho}^{(A)} C_y}{g_{yy}^{(A)}} + \frac{g_{y[\mu}^{(A)} C_{\nu\rho]y}}{g_{yy}^{(A)}} \right),
\end{aligned} \tag{87}$$

where y is the coordinate to which the T dualization is applied, and μ, ν, ρ denote the coordinates other than y . In terms of the T-duality map (87), the solution (85) becomes

$$\begin{aligned}
ds_{(B)}^2 &= h_2^{-1/2}(x, y, z) [h_6(z) h_K(y)]^{-1/4} [\eta_{\mu\nu}(X) dx^\mu dx^\nu + h_2(x, y, z) h_K(y) \delta_{ij}(Y) dy^i dy^j \\
&\quad + h_6(z) h_K(y) dv^2 + h_2(x, y, z) h_6(y) \delta_{ab} dz^a dz^b],
\end{aligned} \tag{88a}$$

$$C_{(2)} = \omega_{(2)}, \tag{88b}$$

$$B_{(2)} = \tilde{\omega}_{(2)}, \tag{88c}$$

$$e^{2\phi^{(B)}} = h_6^{-1} h_K, \tag{88d}$$

$$C_{(4)} = \omega_{(4)} \pm h_2^{-1} \Omega(X) \wedge dv, \tag{88e}$$

where the ten-dimensional line element $ds_{(B)}^2$ is written in the Einstein frame, and the volume 3-form $\Omega(X)$ is defined by (81a), and $C_{(4)}$ and $C_{(2)}$ are gauge potentials for D3- and D5-brane, and $\omega_{(2)}$, $\tilde{\omega}_{(2)}$ and $\omega_{(4)}$ satisfy the relations

$$\begin{aligned}
d\omega_{(2)} &= \partial_a h_6 *_Z dz^a, & d\tilde{\omega}_{(2)} &= \partial_i h_K *_Y dy^i, \\
d\omega_{(4)} &= \pm h_6^{-1} \partial_i h_2 *_Y (dy^i) \pm h_K^{-1} \partial_a h_2 *_Z (dz^a).
\end{aligned} \tag{89}$$

Here $*_Y$ and $*_Z$ are the Hodge operator in the Y and Z space. Finally we obtain the dynamical D3-D5-NS5 solution which is derived from the dynamical D2-D6-KK monopole solution via T-duality.

TABLE VII: Dynamical D3-D5-NS5 brane system. Here \circ denotes the worldvolume coordinate.

Case		0	1	2	3	4	5	6	7	8	9
D3-D5-NS5	D3	\circ	\circ	\circ				\circ			
	D5	\circ	\circ	\circ	\circ	\circ	\circ				
	NS5	\circ	\circ	\circ					\circ	\circ	\circ
	x^N	t	x^1	x^2	y^1	y^2	y^3	v	z^1	z^2	z^3

D. Dynamical M2-pp wave solution

We discuss the dynamical M2-brane and wave solution. We adopt the following ansatz for the eleven-dimensional metric:

$$ds^2 = h_2^{-2/3}(z) \left[-h_w^{-1}(t, y, z) dt^2 + h_w(t, y, z) \{ (h_w^{-1}(t, y, z) - 1) dt + dx \}^2 + dy^2 + h_2(z) u_{ab}(Z) dz^a dz^b \right], \quad (90)$$

where u_{ab} is an eight-dimensional metric which depends only on the eight-dimensional coordinates z^a .

We further require that the 4-form field satisfies the following condition

$$F_{(4)} = d [h_2^{-1}(z)] \wedge dt \wedge dx \wedge dy. \quad (91)$$

TABLE VIII: Dynamical M2-brane and pp wave system in the metric (90). Here \circ denotes the worldvolume coordinate and \star denotes the wave coordinate respectively.

	0	1	2	3	4	5	6	7	8	9	10
M2	\circ	\circ	\circ								
W	\circ	\star									
x^N	t	x	y	z^1	z^2	z^3	z^4	z^5	z^6	z^7	z^8

If we use ansatz for fields (90) and (91), the field equations give

$$R_{ab}(Z) = 0, \quad (92a)$$

$$h_w = h_0(t) + h_1(y, z), \quad \partial_t^2 h_0 = 0, \quad h_2 \partial_y^2 h_1 + \Delta_Z h_1 = 0, \quad \Delta_Z h_2 = 0, \quad (92b)$$

where Δ_Z is the Laplace operator on Z space, and $R_{ab}(Z)$ is the Ricci tensor with respect to the metric $u_{ab}(Z)$. Now we consider the case

$$u_{ab} dz^a dz^b = \delta_{ab} dz^a dz^b = dr^2 + r^2 d\Omega_{(7)}^2, \quad (93)$$

where δ_{ab} is the eight-dimensional Euclidean metrics, and $d\Omega_{(7)}^2$ is the line element of a unit 7-sphere, respectively. The solution for h_2 and h_w can be obtained explicitly as

$$h_w(t, y, r) = \bar{c}t + \tilde{c} + M_w \left(y^2 + \frac{M}{4r^4} \right), \quad (94a)$$

$$h_2(r) = \frac{M}{r^6}, \quad (94b)$$

where \bar{c} , \tilde{c} , M_w and M are constant parameters. If we introduce a coordinate transformation

$$y = \frac{\cos \alpha}{\zeta}, \quad r^2 = \frac{\zeta M^{1/2}}{2 \sin \alpha}, \quad (95)$$

the metric of M2-wave system becomes

$$ds^2 = \frac{M^{1/3}}{4 \sin^2 \alpha} (ds_{\text{AdS}}^2 + d\alpha^2) + M^{1/3} d\Omega_{(7)}^2, \quad (96a)$$

$$ds_{\text{AdS}}^2 = -\zeta^2 h_w^{-1} dt^2 + \zeta^2 h_w [(h_w^{-1} - 1)dt + dx]^2 + \zeta^{-2} d\zeta^2, \quad (96b)$$

$$h_w = At + B + \frac{M_w}{\zeta^2}. \quad (96c)$$

The metric (96) is the extremal BTZ black hole, which is locally AdS_3 . The dynamical M2-pp wave system is a warped product of AdS_3 with an 8-sphere.

E. Cosmological F1-D0 brane solution

In this subsection, we discuss the time dependent F1-D0 brane solution. We assume that the ten-dimensional spacetime has the metric

$$ds^2 = h_F^{-3/4}(z) h^{-7/8}(t, y, z) [-dt^2 + h(t, y, z) dy^2 + h_F(z) h(t, y, z) u_{ab}(Z) dz^a dz^b], \quad (97)$$

where u_{ab} is an eight-dimensional metric which depends only on the eight-dimensional coordinates z^a .

Concerning the other fields, we adopt the following assumptions

$$e^\phi = h_F^{-1/2} h^{3/4}, \quad (98a)$$

$$F_{(2)} = d[h^{-1}(t, y, z)] \wedge dt, \quad (98b)$$

$$H_{(3)} = d[h_F^{-1}(z)] \wedge dt \wedge dy. \quad (98c)$$

Then, the field equations reduce to

$$R_{ab}(Z) = 0, \quad (99a)$$

$$h = h_0(t) + h_1(y, z), \quad h_F = h_F(z), \quad (99b)$$

$$\partial_t^2 h_0 = 0, \quad h_F \partial_y^2 h_1 + \Delta_Z h_1 = 0, \quad \Delta_Z h_F = 0, \quad (99c)$$

TABLE IX: Dynamical F1-D0 brane system in the metric (97). Here \circ denotes the worldvolume coordinate.

	0	1	2	3	4	5	6	7	8	9
F1	\circ	\circ								
D0	\circ									
x^N	t	y	z^1	z^2	z^3	z^4	z^5	z^6	z^7	z^8

where Δ_Z is the Laplace operators on Z space, and $R_{ab}(Z)$ is the Ricci tensor constructed from the metric $u_{ab}(Z)$, respectively. We assume that the metric of Z space is given by

$$u_{ab} = \delta_{ab}, \quad (100)$$

where δ_{ab} is the eight-dimensional Euclidean metrics. Thus, the solution of h and h_F can be expressed as

$$h(t, y, z) = \bar{c}t + \tilde{c} + \sum_{\ell} M_{\ell} \left[(y - y_{\ell})^2 + \frac{M_F}{4|z^a - z_0^a|^4} \right], \quad (101a)$$

$$h_F(z) = \frac{M_F}{|z^a - z_0^a|^6}, \quad (101b)$$

where \bar{c} , \tilde{c} , M_{ℓ} and M_F are constants, and y_{ℓ} and z_0^a are also constant parameters.

We consider the case where the D0-brane is located at the origin of the Y space and use the coordinate

$$\delta_{ab} dz^a dz^b = dr^2 + r^2 d\Omega_{(7)}^2, \quad (102)$$

where $d\Omega_{(7)}^2$ is the line element of seven-dimensional unit 7-sphere. Then we have

$$h(x, r) = A_{\mu} x^{\mu} + B + M \left(y^2 + \frac{M_F}{4r^4} \right), \quad h_F(r) = \frac{M_F}{r^6}, \quad (103)$$

where M is constant.

If we use a coordinate transformation

$$y = \frac{\cos \alpha}{\zeta}, \quad r^2 = \frac{\zeta M_F^{1/2}}{2 \sin \alpha}, \quad (104)$$

the metric of the near-horizon region is written by

$$ds^2 = 8^{-3/4} M^{1/8} M_F^{3/8} (\sin \alpha)^{-9/4} \left(-\frac{\zeta^4}{M} dt^2 + \frac{d\zeta^2}{\zeta^2} + d\alpha^2 + 4 \sin^2 \alpha d\Omega_{(7)}^2 \right), \quad (105)$$

where $d\Omega_{(7)}^2$ is the line element of a unit 7-sphere. This is a metric of warped product of AdS_2 with an 8-sphere.

F. Dynamical M2-M2 brane solution

In this subsection, we discuss the dynamical intersecting M2 brane solution. We adopt the metric ansatz for the eleven-dimensional spacetime

$$ds^2 = h_2^{1/3}(t, x, z) k_2^{1/3}(z) \left[-h_2^{-1}(t, x, z) k_2^{-1}(z) dt^2 + k_2^{-1}(z) q_{\mu\nu}(X) dx^\mu dx^\nu \right. \\ \left. + h_2^{-1}(t, x, z) \gamma_{ij}(Y) dy^i dy^j + u_{ab}(Z) dz^a dz^b \right], \quad (106)$$

where $q_{\mu\nu}$ is a two-dimensional metric which depends only on the two-dimensional coordinates y^i , γ_{ij} is a two-dimensional metric which depends only on the two-dimensional coordinates y^i , and finally u_{ab} is a six-dimensional metric which depends only on the six-dimensional coordinates z^a .

The 4-form gauge field strength $F_{(4)}$ is assumed to be

$$F_{(4)} = d \left[h_2^{-1}(t, x, z) dt \wedge \Omega(Y) + k_2^{-1}(z) dt \wedge \Omega(X) \right], \quad (107)$$

where $\Omega(X)$ and $\Omega(Y)$ denote the volume 2-forms

$$\Omega(X) = \sqrt{q} dx^1 \wedge dx^2, \quad (108a)$$

$$\Omega(Y) = \sqrt{\gamma} dy^1 \wedge dy^2. \quad (108b)$$

Here, q and γ are the determinants of the metric $q_{\mu\nu}$, and γ_{ij} , respectively.

TABLE X: Dynamical M2-M2 brane system in the metric (106). Here \circ denotes the worldvolume coordinate.

	0	1	2	3	4	5	6	7	8	9	10
M2	\circ	\circ	\circ								
M2	\circ			\circ	\circ						
x^N	t	x^1	x^2	y^1	y^2	z^1	z^2	z^3	z^4	z^5	z^6

Under the assumption for the metric (106) and field strength (107), the field equations give

$$R_{\mu\nu}(X) = 0, \quad R_{ij}(Y) = 0, \quad R_{ab}(Z) = 0, \quad (109a)$$

$$h_2 = h_0(t) + h_1(x, z), \quad \partial_t^2 h_0 = 0, \quad k_2 \Delta_X h_1 + \Delta_Z h_1 = 0, \quad \Delta_Z k_2 = 0, \quad (109b)$$

where Δ_X , Δ_Z are the Laplace operators on the space of X, Z, and $R_{\mu\nu}(X)$, $R_{ij}(Y)$ and $R_{ab}(Z)$ are the Ricci tensors with respect to the metrics $q_{\mu\nu}(X)$, $\gamma_{ij}(Y)$, $u_{ab}(Z)$, respectively. Now we consider the case

$$q_{\mu\nu} = \delta_{\mu\nu}, \quad \gamma_{ij} = \delta_{ij}, \quad u_{ab} = \delta_{ab}, \quad (110)$$

where $\delta_{\mu\nu}$, δ_{ij} , δ_{ab} are the two-, two-, six-dimensional Euclidean metrics, respectively. The solution for h_2 and k_2 can be obtained explicitly as

$$h_2(t, x, z) = \bar{c}t + \tilde{c} + \sum_{\ell} M_{\ell} [|x^{\mu} - x_{\ell}^{\mu}|^2 + M_2 |z^a - z_0^a|^{-2}], \quad (111a)$$

$$k_2(z) = \frac{M_2}{|z^a - z_0^a|^4}, \quad (111b)$$

where \bar{c} , \tilde{c} , M_{ℓ} and M_2 are constants, and the constant x_{ℓ}^{μ} and z_0^a represent the positions of the branes. Now we consider the case where M2-brane is located at the origin of the X and Z spaces. If we use the coordinate transformation

$$x^1 = \frac{1}{r} \cos \theta \cos \alpha, \quad x^2 = \frac{1}{r} \sin \theta \cos \alpha, \quad z^a = \frac{r M_2^{1/2}}{\sin \alpha} \mu^a, \quad (112)$$

the harmonic functions take the following form:

$$h_2(t, r) = \bar{c}t + \tilde{c} + \frac{M}{r^2}, \quad (113a)$$

$$k_2(r) = \frac{\sin^4 \alpha}{M_2 r^4}, \quad (113b)$$

where M is constant, and μ^a is defined as

$$\mu_a \mu^a = 1, \quad d\Omega_{(5)}^2 = d\mu_a d\mu^a. \quad (114)$$

Here $d\Omega_{(5)}^2$ is the line element of the unit 5-sphere.

The eleven-dimensional metric in the near horizon limit $r \rightarrow 0$ is thus written by

$$ds^2 = M^{1/3} M_2^{2/3} (\sin \theta)^{-8/3} \left[-\frac{r^4}{M} dt^2 + \frac{dr^2}{r^2} + d\alpha^2 + \cos^2 \theta d\theta^2 \right. \\ \left. + \sin^2 \theta d\Omega_{(5)}^2 + (M M_2)^{-1} \sin^4 \theta \left\{ (dy^1)^2 + (dy^2)^2 \right\} \right], \quad (115)$$

where $d\Omega_{(5)}^2$ is the line element of a unit 5-sphere. The near-horizon geometry of this system is a warped product of AdS_2 with a 7-sphere and a 2-torus. We can obtain the solution (113) in terms of lifting the F1-D2 brane solution (42) back to the eleven-dimensional theory.

If we replace the 5-sphere with a lens space in the eleven-dimensional metric (115), we can obtain the dynamical solutions M2-M2-KK monopole system. The gauge field strength $F_{(4)}$ is given by (107). The solution is written by

$$ds^2 = h_2^{1/3}(t, x, \ell, r) k_2^{1/3}(r) \left[-h_2^{-1}(t, x, \ell, r) k_2^{-1}(\ell, r) dt^2 + k_2^{-1}(\ell, r) \delta_{\mu\nu} dx^\mu dx^\nu + \delta_{mn} d\ell^m d\ell^n \right. \\ \left. + h_2^{-1}(t, x, \ell, r) \delta_{ij} dy^i dy^j + h_K(r) \delta_{ab} dz^a dz^b + h_K^{-1}(r) (dv + A_a dz^a)^2 \right], \quad (116a)$$

$$\delta_{ab}(Z) dz^a dz^b = dr^2 + r^2 d\Omega_{(2)}^2, \quad (116b)$$

where $\delta_{\mu\nu}$, δ_{mn} , δ_{ij} , δ_{ab} are the two-, two-, two-, three-dimensional flat metrics, and finally $d\Omega_{(2)}^2$ is the metric of a 2-sphere, respectively. The functions h_2 , k_2 , and h_K are written by

$$h_2(t, x, \ell, r) = \bar{c}t + \tilde{c} + \sum_l M_l \left[|x^\mu - x_l^\mu|^2 + \frac{M_2}{|\ell^m - \ell_0^m|^2 + 4M_K r} \right], \quad (117a)$$

$$k_2(\ell, r) = \frac{M_2}{(|\ell^m - \ell_0^m|^2 + 4M_K r)^2}, \quad (117b)$$

$$h_K(r) = \frac{M_K}{r}, \quad (117c)$$

where \bar{c} , \tilde{c} , M_l , M_2 and M_K are constant parameters, and the constant x_l^μ , ℓ_0^m denote the positions of the branes.

TABLE XI: Time dependent M2-M2 KK monopole in the metric (116). Here \circ denotes the world-volume coordinate and \bullet denotes the fibre coordinate of the KK-monopole respectively.

	0	1	2	3	4	5	6	7	8	9	10
M2	\circ	\circ	\circ								
M2	\circ			\circ	\circ						
KK	\circ	\circ	\circ	\circ	\circ	\circ	\circ	\bullet			
x^N	t	x^1	x^2	y^1	y^2	ℓ^1	ℓ^2	v	z^1	z^2	z^3

G. Dynamical D2-D2-D6 brane solution

Now we construct the dynamical D2-D2-D6 brane solutions in ten-dimensional type IIA string theory.

Let us consider the ten-dimensional spacetime with the metric

$$\begin{aligned}
ds^2 = & h_2^{3/8}(t, x, \ell, z) k_2^{3/8}(\ell, z) h_6^{7/8}(z) \left[-h_2^{-1}(t, x, \ell, z) k_2^{-1}(\ell, z) h_6^{-1}(z) dt^2 \right. \\
& + k_2^{-1}(\ell, z) h_6^{-1}(z) q_{\mu\nu}(X) dx^\mu dx^\nu + h_2^{-1}(t, x, \ell, z) h_6^{-1}(z) \gamma_{ij}(Y) dy^i dy^j \\
& \left. + h_6^{-1}(z) w_{mn}(L) d\ell^m d\ell^n + u_{ab}(Z) dz^a dz^b \right], \tag{118}
\end{aligned}$$

where $q_{\mu\nu}$ is a two-dimensional metric which depends only on the two-dimensional coordinates x^μ , γ_{ij} is a two-dimensional metric which depends only on the two-dimensional coordinates y^i , w_{mn} is a two-dimensional metric which depends only on the two-dimensional coordinates ℓ^m , and finally u_{ab} is a three-dimensional metric which depends only on the three-dimensional coordinates z^a . We further require that the scalar field ϕ and the form fields satisfy the following conditions:

$$e^\phi = (h_2 k_2)^{1/4} h_6^{-3/4}, \tag{119a}$$

$$F_{(2)} = e^{-3\phi/2} * [d(h_6^{-1}) \wedge \Omega(X) \wedge \Omega(Y) \wedge \Omega(L)], \tag{119b}$$

$$F_{(4)} = d(h_2^{-1}) \wedge dt \wedge \Omega(Y) + d(k_2^{-1}) \wedge dt \wedge \Omega(X), \tag{119c}$$

where $\Omega(X)$, $\Omega(Y)$ and $\Omega(L)$ denote the volume 2-forms,

$$\Omega(X) = \sqrt{q} dx^1 \wedge dx^2, \tag{120a}$$

$$\Omega(Y) = \sqrt{\gamma} dy^1 \wedge dy^2, \tag{120b}$$

$$\Omega(L) = \sqrt{w} d\ell^1 \wedge d\ell^2, \tag{120c}$$

respectively.

TABLE XII: Dynamical D2-D2-D6 brane system in the metric (118). Here \circ denotes the world-volume coordinate.

	0	1	2	3	4	5	6	7	8	9
D2	\circ	\circ	\circ							
D2	\circ			\circ	\circ					
D6	\circ	\circ	\circ	\circ	\circ	\circ	\circ			
x^N	t	x^1	x^2	y^1	y^2	ℓ^1	ℓ^2	z^1	z^2	z^3

The field equations reduce to

$$R_{\mu\nu}(X) = 0, \quad R_{ij}(Y) = 0, \quad R_{mn}(L) = 0, \quad R_{ab}(Z) = 0, \quad (121a)$$

$$h_2(t, x, \ell, z) = K_0(t) + K_1(x, \ell, z), \quad k_2 = k_2(\ell, z), \quad h_6 = h_6(z);$$

$$\partial_t^2 K_0 = 0, \quad \Delta_Z K_1 + h_6 (\Delta_L K_1 + k_2 \Delta_X K_1) = 0,$$

$$\Delta_Z k_2 + h_6 \Delta_L k_2 = 0, \quad \Delta_Z h_6 = 0, \quad (121b)$$

where $\Delta_X, \Delta_L, \Delta_Z$ are the Laplace operators on X, L, Z space, and $R_{\mu\nu}(X), R_{ij}(Y), R_{mn}(L)$ and $R_{ab}(Z)$ are the Ricci tensors constructed from the metrics $q_{\mu\nu}(X), \gamma_{ij}(Y), w_{mn}(L), u_{ab}(Z)$, respectively. As a special example, we consider the case

$$q_{\mu\nu} = \delta_{\mu\nu}, \quad \gamma_{ij} = \delta_{ij}, \quad u_{ab} = \delta_{ab}, \quad w_{mn} = \delta_{mn}, \quad (122)$$

where $\delta_{\mu\nu}, \delta_{ij}, \delta_{mn}$, and δ_{ab} are the two-, two-, two-, three-dimensional Euclidean metric. In this case, the solution of field equations can be obtained explicitly as

$$h_2(t, x, \ell, z) = \bar{c}t + \tilde{c} + \sum_l M_l \left[|x^\mu - x_l^\mu|^2 + \frac{M_2}{|\ell^m - \ell_0^m|^2 + 4M_6|z^a - z_0^a|} \right], \quad (123a)$$

$$k_2(\ell, z) = \frac{M_2}{(|\ell^m - \ell_0^m|^2 + 4M_6|z^a - z_0^a|)^2}, \quad (123b)$$

$$h_6(z) = \frac{M_6}{|z^a - z_0^a|}, \quad (123c)$$

where $\bar{c}, \tilde{c}, M_l, M_2$ and M_6 are constant parameters, and the constants x_l^μ, ℓ_0^m and z_0^a denote the positions of the branes.

If we perform Kaluza-Klein reduction on the fibre coordinate v in the M2-M2-KK monopole system (116), we can construct the dynamical D2-D2-D6 brane solution.

IV. ADS SPACETIME FROM M5-BRANE SOLUTIONS

In this section, we discuss the dynamical intersecting brane solutions including M5-waves and KK monopole in eleven dimensions. The dimensional reduction of these generates the cosmological D-brane solutions in the ten-dimensional supergravity theories. We also briefly discuss those objects.

A. Dynamical M5-brane and KK monopole solution

In this subsection, we construct the time dependent M5-brane and KK monopole solution. The eleven-dimensional metric is assumed to be [30]

$$ds^2 = h_5^{-1/3}(x, y, z) q_{\mu\nu}(X) dx^\mu dx^\nu + h_5^{2/3}(x, y, z) [dy^2 + h_K(z) u_{ab}(Z) dz^a dz^b + h_K^{-1}(z) (dv + A_a dz^a)^2], \quad (124a)$$

$$u_{ab}(Z) dz^a dz^b = dr^2 + r^2 w_{mn}(Z') dp^m dp^n, \quad (124b)$$

where $q_{\mu\nu}$ is a six-dimensional metric which depends only on the six-dimensional coordinates x^μ , w_{mn} is a two-dimensional metric which depends only on the two-dimensional coordinates p^m , and finally u_{ab} is a three-dimensional metric which depends only on the three-dimensional coordinates z^a .

We also assume the form of 4-form gauge field strength $F_{(4)}$;

$$F_{(4)} = *d [h_5^{-1}(x, y, z) \wedge \Omega(X)], \quad (125)$$

where $\Omega(X)$ is given by

$$\Omega(X) = \sqrt{-q} dx^0 \wedge dx^1 \wedge \cdots \wedge dx^5. \quad (126)$$

Here, q is the determinant of the metric $q_{\mu\nu}$.

TABLE XIII: Dynamical M5-brane and KK monopole system in the metric (124a). Here \circ denotes the worldvolume coordinate and \bullet denotes the fibre coordinate of the KK-monopole respectively.

	0	1	2	3	4	5	6	7	8	9	10
M5	\circ	\circ	\circ	\circ	\circ	\circ					
KK	\circ	\circ	\circ	\circ	\circ	\circ	\circ	\bullet			
x^N	t	x^1	x^2	x^3	x^4	x^5	y	v	z^1	z^2	z^3

With ansatz for fields (124) and (125), the field equations reduce to

$$R_{\mu\nu}(X) = 0, \quad R_{ab}(Z) = 0, \quad R_{mn}(Z') = w_{mn}(Z'), \quad (127a)$$

$$h_5 = h_0(x) + h_1(y, z), \quad dh_K = *_Z dA_a, \quad (127b)$$

$$D_\mu D_\nu h_0 = 0, \quad h_K \partial_y^2 h_1 + \triangle_Z h_1 = 0, \quad \triangle_Z h_K = 0, \quad (127c)$$

where D_μ is the covariant derivative with respect to the metric $q_{\mu\nu}$, and Δ_Z is the Laplace operator on Z space, and $R_{\mu\nu}(X)$, $R_{ab}(Z)$, $R_{mn}(Z')$ are the Ricci tensors with respect to the metrics $q_{\mu\nu}(X)$, $u_{ab}(Z)$, $w_{mn}(Z')$, respectively.

To see the solutions more explicitly, we consider the case of

$$q_{\mu\nu} = \eta_{\mu\nu}, \quad u_{ab}dz^a dz^b = \delta_{ab}dz^a dz^b = dr^2 + r^2 d\Omega_{(2)}^2, \quad (128)$$

where $\eta_{\mu\nu}$ is the six-dimensional Minkowski metric and δ_{ab} is the three-dimensional Euclidean metric, and $d\Omega_{(2)}^2$ is the line element of 2-sphere, respectively. Then, the solution for h_5 and h_K can be written as

$$h_5(x, y, r) = c_\mu x^\mu + \tilde{c} + \sum_\ell \frac{M_\ell}{(|y - y_\ell|^2 + 4M_K r)^{3/2}}, \quad (129a)$$

$$h_K(z) = \frac{M_K}{r}, \quad (129b)$$

where c_μ , \tilde{c} , M_ℓ and M_K are constant parameters, and the constant y_ℓ represents the position of the brane.

Now we consider the case where the M5-brane is located at the origin of the y and make a change of coordinates

$$y = \zeta \cos \alpha, \quad r = \frac{1}{4} M_K^{-1} \zeta^2 \sin^2 \alpha. \quad (130)$$

In terms of (130) and $y_\ell = 0$, the eleven-dimensional metric (124) reads

$$ds^2 = h_5^{-2/3} \eta_{\mu\nu}(X) dx^\mu dx^\nu + h_5^{-2/3} [d\zeta^2 + \zeta^2 ds^2(M_4)], \quad (131)$$

where h_5 and $ds^2(M_4)$ are given by

$$h_5 = c_\mu x^\mu + \tilde{c} + \frac{M_5}{\zeta^3}, \quad (132)$$

$$ds^2(M_4) = d\alpha^2 + \frac{1}{4} \sin^2 \alpha [d\Omega_{(2)}^2 + (dv + A_a dz^a)^2], \quad (133)$$

where M_5 is constant. In the near horizon limit $\zeta \rightarrow 0$, the metric becomes $\text{AdS}_7 \times M_4$.

B. Dynamical D6-NS5 brane solution

Now we construct the dynamical D6-NS5 branes system. In this subsection, we look for solutions whose ten-dimensional metrics have the form

$$ds^2 = h^{7/8}(z) h_{\text{NS}}^{3/4}(x, y, z) \left[h^{-1}(z) h_{\text{NS}}^{-1}(x, y, z) q_{\mu\nu}(X) dx^\mu dx^\nu + h^{-1}(z) dy^2 + u_{ab}(Z) dz^a dz^b \right], \quad (134a)$$

$$u_{ab}(Z) dz^a dz^b = dr^2 + r^2 w_{mn}(Z') dp^m dp^n, \quad (134b)$$

where $q_{\mu\nu}$ is a six-dimensional metric which depends only on the six-dimensional coordinates x^μ , w_{mn} is a two-dimensional metric which depends only on the two-dimensional coordinates p^m , and u_{ab} is a three-dimensional metric which depends only on the three-dimensional coordinates z^a .

We also assume that other fields are the function of time

$$e^\phi = h_{\text{NS}}^{1/2} h^{-3/4}, \quad (135a)$$

$$H_{(3)} = e^\phi * d \left[h_{\text{NS}}^{-1}(x, y, z) \Omega(X) \right], \quad (135b)$$

$$F_{(2)} = e^{-3\phi/2} * d \left[h^{-1}(z) \Omega(X) \wedge dy \right], \quad (135c)$$

where $\Omega(X)$ denotes the volume 6-form

$$\Omega(X) = \sqrt{-q} dx^0 \wedge dx^1 \wedge \cdots \wedge dx^5. \quad (136)$$

Here, q is the determinant of the metric $q_{\mu\nu}$.

TABLE XIV: Dynamical D6-NS5 brane system in the metric (134a). Here \circ denotes the worldvolume coordinate.

	0	1	2	3	4	5	6	7	8	9
D6	\circ	\circ	\circ	\circ	\circ	\circ	\circ			
NS5	\circ	\circ	\circ	\circ	\circ	\circ				
x^N	t	x^1	x^2	x^3	x^4	x^5	y	z^1	z^2	z^3

From the ansatz for fields (134) and (135), we get

$$R_{\mu\nu}(X) = 0, \quad R_{ab}(Z) = 0, \quad R_{mn}(Z') = w_{mn}(Z'), \quad (137a)$$

$$h_{\text{NS}} = h_0(x) + h_1(y, r), \quad D_\mu D_\nu h_0 = 0, \quad h \partial_y^2 h_1 + \Delta_Z h_1 = 0, \quad \Delta_Z h = 0, \quad (137b)$$

where D_μ is the covariant derivative constructed by the metric $q_{\mu\nu}$, and Δ_Z is the Laplace operator on Z space, and $R_{\mu\nu}(X)$, $R_{ab}(Z)$ and $R_{mn}(Z')$ are the Ricci tensors with respect to the metrics $q_{\mu\nu}(X)$, $u_{ab}(Z)$, $w_{mn}(Z')$, respectively. Let us consider the case

$$q_{\mu\nu} = \eta_{\mu\nu}, \quad u_{ab} = \delta_{ab} = \delta_{ab} dz^a dz^b = dr^2 + r^2 d\Omega_{(2)}^2, \quad (138)$$

where $\eta_{\mu\nu}$ is the six-dimensional Minkowski metric and δ_{ab} is the three-dimensional Euclidean metric, and $d\Omega_{(2)}^2$ is the line element of 2-sphere, respectively, respectively. The solution for h and h_{NS} can be obtained explicitly as

$$h_{\text{NS}}(x, y, r) = c_\mu x^\mu + \tilde{c} + \sum_\ell \frac{M_\ell}{[|y - y_\ell|^2 + 4Mr]^{\frac{3}{2}}}, \quad (139a)$$

$$h(r) = \frac{M}{r}, \quad (139b)$$

where c_μ , \tilde{c} , M_ℓ and M are constant parameters, and the constant y_ℓ denotes the position of the brane. The solution (134) and (139) can be obtained by the dimensional reduction on the direction of v in the solution (124). If we consider the case where D6-brane is located at the origin of the Y , Z spaces and use the coordinate transformation,

$$y = \zeta \cos \alpha, \quad r = \frac{1}{4} M^{-1} \zeta^2 \sin^2 \alpha, \quad (140)$$

we have

$$h_{\text{NS}}(x, \zeta) = c_\mu x^\mu + \tilde{c} + \frac{M_5}{\zeta^3}, \quad (141a)$$

$$h(\zeta) = \frac{4M^2}{\zeta^2 \sin^2 \alpha}. \quad (141b)$$

In the near horizon limit $\zeta \rightarrow 0$, the metric becomes

$$ds^2 = M^{3/4} (2M_5)^{-1/4} (\sin \alpha)^{1/4} \left[-\frac{\zeta}{M} \eta_{\mu\nu}(X) dx^\mu dx^\nu + \frac{d\zeta^2}{\zeta^2} + d\alpha^2 + \frac{1}{4} \sin^2 \alpha d\Omega_{(2)}^2 \right], \quad (142)$$

where $d\Omega_{(2)}^2$ is the line element of a unit 2-sphere. Then, the ten-dimensional metric is a warped product of the AdS_7 with a three-dimensional internal space.

We can construct the solution (141) after dimensional reduction on the fibre coordinate v in the M5-KK monopole (129). For $c_\mu = 0$, the function (139a) is the consistent with the static D6-NS5 solution [36].

C. Dynamical M5-brane and pp-wave solution

We construct the dynamical M5-brane and pp-wave system. In this subsection, we take the following metric ansatz

$$ds^2 = h_5^{2/3}(z) \left[-h_5^{-1}(z)h_w^{-1}(t, y, z)dt^2 + h_5^{-1}(z)h_w(t, y, z) \left\{ (h_w^{-1}(t, y, z) - 1) dt + dx \right\}^2 + h_5^{-1}(z)\gamma_{ij}(Y)dy^i dy^j + u_{ab}(Z)dz^a dz^b \right], \quad (143)$$

where γ_{ij} is a four-dimensional metric which depends only on the four-dimensional coordinates y^i , and finally u_{ab} is a five-dimensional metric which depends only on the five-dimensional coordinates z^a .

We now take the following ansatz for the gauge field strength $F_{(4)}$

$$F_{(4)} = *d \left[h_5^{-1}(z) \wedge dt \wedge dx \wedge \Omega(Y) \right], \quad (144)$$

where the volume 4-form $\Omega(Y)$ is

$$\Omega(Y) = \sqrt{\gamma} dy^1 \wedge dy^2 \wedge dy^3 \wedge dy^4. \quad (145)$$

Here, γ is the determinant of the metric γ_{ij} .

TABLE XV: Dynamical M5-brane pp wave system in the metric (143). Here \circ denotes the world-volume coordinate and \star denotes the wave coordinate respectively.

	0	1	2	3	4	5	6	7	8	9	10
M5	\circ	\circ	\circ	\circ	\circ	\circ					
W	\circ	\star									
x^N	t	x	y^1	y^2	y^3	y^4	z^1	z^2	z^3	z^4	z^5

By using the ansatz for fields (143) and (144), the field equations reduce to

$$R_{ij}(Y) = 0, \quad R_{ab}(Z) = 0, \quad (146a)$$

$$h_w = h_0(t) + h_1(y, z), \quad \partial_t^2 h_0 = 0, \quad h_5 \Delta_Y h_1 + \Delta_Z h_1 = 0, \quad \Delta_Z h_5 = 0, \quad (146b)$$

where Δ_Y , Δ_Z are the Laplace operators on Y , Z space, and $R_{ij}(Y)$ and $R_{ab}(Z)$ are the Ricci tensors with respect to the metrics $\gamma_{ij}(Y)$, $u_{ab}(Z)$, respectively. We take the metric on Y and Z space to be

$$\gamma_{ij} = \delta_{ij}, \quad u_{ab} dz^a dz^b = \delta_{ab} dz^a dz^b = dr^2 + r^2 d\Omega_{(4)}^2, \quad (147)$$

where δ_{ij} , δ_{ab} are the four-dimensional Euclidean metrics, and $d\Omega_{(4)}^2$ is the line element of a unit 4-sphere, respectively. Then we can get solution for h_5 and h_w

$$h_w(t, y, r) = \bar{c}t + \tilde{c} + M_w \left(y^2 + \frac{4M}{r} \right), \quad (148a)$$

$$h_5(r) = \frac{M}{r^3}, \quad (148b)$$

where $y^2 = \delta_{ij}y^iy^j$, and \bar{c} , \tilde{c} , M_w and M are constant parameters.

If we introduce coordinate transformation

$$y^i = \frac{\mu^i}{\zeta} \cos \alpha, \quad r = \frac{4\zeta^2 M}{\sin^2 \alpha}, \quad (149)$$

the metric of the M5-brane and wave system becomes

$$ds^2 = \frac{4M^{2/3}}{\sin^2 \alpha} (ds_{\text{AdS}}^2 + d\alpha^2 + \cos^2 \alpha d\Omega_{(3)}^2) + M^{2/3} d\Omega_{(4)}^2, \quad (150a)$$

$$ds_{\text{AdS}}^2 = -\zeta^2 h_w^{-1} dt^2 + \zeta^2 h_w [(h_w^{-1} - 1)dt + dx]^2 + \zeta^{-2} d\zeta^2, \quad (150b)$$

$$h_w = At + B + \frac{M_w}{\zeta^2}, \quad (150c)$$

where $\mu^i \mu_i = 1$, and the line element of the 3-sphere is defined by $d\mu^i d\mu_i = d\Omega_{(3)}^2$. The metric (150) is the extremal BTZ black hole, which is locally AdS_3 . The dynamical M5-pp wave system is a warped product of AdS_3 with the internal spaces.

If we perform the dimensional reduction of the solution (148) on the coordinate x , the M5-pp wave becomes D0-D4 brane system. We will discuss the dynamical D0-D4 brane solution in the next subsection.

D. Dynamical D0-D4 brane solution

In this subsection, we discuss the dynamical solution of D0-D4 brane system. We assume that the ten-dimensional metric is written by

$$ds^2 = h^{1/8}(t, y, z) h_4^{5/8}(z) [-h^{-1}(t, y, z) h_4^{-1}(z) dt^2 + h_4^{-1}(z) \gamma_{ij}(Y) dy^i dy^j + u_{ab}(Z) dz^a dz^b], \quad (151)$$

where γ_{ij} is a four-dimensional metric which depends only on the four-dimensional coordinates y^i , and finally u_{ab} is a five-dimensional metric which depends only on the five-dimensional coordinates z^a .

TABLE XVI: Dynamical D0-D4 brane system. Here \circ denotes the worldvolume coordinate.

Case		0	1	2	3	4	5	6	7	8	9
D0-D4	D0	\circ									
	D4	\circ	\circ	\circ	\circ	\circ					
	x^N	t	y^1	y^2	y^3	y^4	z^1	z^2	z^3	z^4	z^5

The scalar field ϕ and the gauge field strengths $F_{(2)}$, $F_{(4)}$ are assumed to be

$$e^\phi = h^{3/4} h_4^{-1/4}, \quad (152a)$$

$$F_{(2)} = d[h^{-1}(x, y, z)] \wedge dt, \quad (152b)$$

$$F_{(4)} = e^{-\phi/2} * d[h_4^{-1}(z) dt \wedge \Omega(Y)], \quad (152c)$$

where the volume 4-form $\Omega(Y)$ is given by

$$\Omega(Y) = \sqrt{\gamma} dy^1 \wedge dy^2 \wedge dy^3 \wedge dy^4. \quad (153)$$

Here, γ are the determinant of the metric γ_{ij} .

Under the ansatz for fields (151) and (152), the field equations reduce to

$$R_{ij}(Y) = 0, \quad R_{ab}(Z) = 0, \quad (154a)$$

$$h = K_0(t) + K_1(y, z), \quad \partial_t^2 K_0 = 0, \quad h_4 \Delta_Y K_1 + \Delta_Z K_1 = 0, \quad \Delta_Z h_4 = 0, \quad (154b)$$

where Δ_Y , Δ_Z are the Laplace operators on Y , Z space, and $R_{ij}(Y)$ and $R_{ab}(Z)$ are the Ricci tensors constructed from the metrics $\gamma_{ij}(Y)$, $u_{ab}(Z)$, respectively.

Now we consider the case

$$\gamma_{ij} = \delta_{ij}, \quad u_{ab} = \delta_{ab}, \quad (155)$$

where δ_{ij} , δ_{ab} are the four-, five-dimensional Euclidean metrics, respectively. Then, the functions h_0 and h_4 are

$$h(t, y, z) = ct + \tilde{c} + \sum_{\ell} M_{\ell} [|y^i - y_{\ell}^i|^2 + 4M_4 |z^a - z_0^a|^{-1}], \quad (156a)$$

$$h_4(z) = \frac{M_4}{|z^a - z_0^a|^3}, \quad (156b)$$

where c , \tilde{c} , M_{ℓ} and M_4 are constant parameters, and y_{ℓ}^i and z_0^a are also constants. Now we use the metric

$$u_{ab} dz^a dz^b = \delta_{ab} dz^a dz^b = dr^2 + r^2 d\Omega_{(4)}^2, \quad (157)$$

where $d\Omega_{(4)}^2$ is the line element of a unit 4-sphere, respectively. The forms of h and h_4 are thus replaced with

$$h(t, y, z) = ct + \tilde{c} + M \left(y^2 + \frac{4M_4}{r} \right), \quad (158a)$$

$$h_4(z) = \frac{M_4}{r^3}, \quad (158b)$$

where the D0-brane is located at the origin of the Y space, and M is constant. In terms of Eqs.(157), (158) and coordinate transformation

$$y^i = \frac{\mu^i}{\zeta} \cos \alpha, \quad r = \frac{4\zeta^2 M_4}{\sin^2 \alpha}, \quad (159)$$

the metric in the near horizon limit is written by

$$ds^2 = \frac{2^{9/4} M^{1/8} M_4^{3/4}}{(\sin \alpha)^{9/4}} \left(-\frac{\zeta^4}{M} dt^2 + \frac{d\zeta^2}{\zeta^2} + d\alpha^2 + \cos^2 \alpha d\Omega_{(3)}^2 + \frac{1}{4} \sin^2 \alpha d\Omega_{(4)}^2 \right), \quad (160)$$

where $\mu^i \mu_i = 1$, and the line element of the 3-sphere is defined by $d\mu^i d\mu_i = d\Omega_{(3)}^2$. Thus, the ten-dimensional metric (151) describes that the dynamical D0-D4 brane system is a warped product of AdS_2 with the internal spaces.

V. ASYMPTOTIC ADS SPACETIME IN MASSIVE IIA THEORY

We have so far the examples of intersecting Dp - $D(p+4)$ ($p = 0, 2$) systems in the type IIA theory that give rise to warped products of AdS_{p+2} with certain internal spaces. In this section, we consider dynamical solutions for the D4-D8 brane system, which appears in the ten-dimensional massive type IIA supergravity. The bosonic action of D4-D8 brane system in the Einstein frame is given by [25, 30–33]

$$S = \frac{1}{2\kappa^2} \int \left(R * \mathbf{1} - \frac{1}{2} d\phi \wedge * d\phi - \frac{1}{2 \cdot 4!} e^{\phi/2} F_{(4)} \wedge * F_{(4)} - \frac{1}{2} e^{5\phi/2} m^2 * \mathbf{1} \right), \quad (161)$$

where κ^2 is the ten-dimensional gravitational constant, $*$ is the Hodge dual operator in the ten-dimensional spacetime, and m is a constant parameter, which is the dual of the 10-form field strength $F_{(10)}$ in the string frame.

The field equations are

$$d * d\phi = \frac{1}{4} \left(5m^2 e^{5\phi/2} * \mathbf{1} + \frac{1}{4!} e^{\phi/2} F_{(4)} \wedge * F_{(4)} \right), \quad (162a)$$

$$d(e^{\phi/2} * F_{(4)}) = 0, \quad (162b)$$

$$R_{MN} = \frac{1}{2} \partial_M \phi \partial_N \phi + \frac{1}{16} m^2 e^{5\phi/2} g_{MN} + \frac{1}{2 \cdot 4!} e^{\phi/2} \left(4F_{MABC} F_N{}^{ABC} - \frac{3}{8} g_{MN} F_{(4)}^2 \right). \quad (162c)$$

It was observed that we can construct a solution whose spacetime metric has the form [8]

$$ds^2 = h^{1/12} \left[h_4^{-3/8} q_{\mu\nu}(X) dx^\mu dx^\nu + h_4^{5/8} (dr^2 + r^2 d\Omega_{(4)}^2) \right], \quad (163a)$$

$$d\Omega_{(4)}^2 = d\alpha^2 + \cos^2 \alpha d\Omega_{(3)}^2, \quad (163b)$$

$$h_4(x, r) = a_\mu x^\mu + \frac{c_1}{r^{10/3}} + c_2, \quad h(r, \alpha) = \frac{3}{2} m r \sin \alpha, \quad (163c)$$

where $q_{\mu\nu}$ is a five-dimensional metric depending only on the coordinates x^μ of X , and $d\Omega_{(3)}^2$ and $d\Omega_{(4)}^2$ denote the line elements of unit 3- and 4-spheres, and a_μ , c_1 and c_2 are constants, respectively. As for the scalar field and the 4-form field strength, we can adopt the following forms:

$$e^\phi = h^{-5/6} h_4^{-1/4}, \quad (164a)$$

$$F_{(4)} = e^{-\phi/2} * [d(h_4^{-1}) \wedge \Omega(X)], \quad (164b)$$

where $\Omega(X)$ is given by

$$\Omega(X) = \sqrt{-q} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4. \quad (165)$$

If we further define a new coordinate U by $r^2 = U^3$. From Eq. (163c), we see that h_4 is a linear function of x^μ . Hence, keeping the values of these coordinates finite, the metric in the limit $U \rightarrow 0$ becomes

$$ds^2 = c_1^{1/8} \left(\frac{3}{2} m \sin \alpha \right)^{1/12} \left[c_1^{-1/2} U^2 q_{\mu\nu} dx^\mu dx^\nu + c_1^{1/2} \left(\frac{9dU^2}{4U^2} + d\Omega_{(4)}^2 \right) \right], \quad (166)$$

while the dilaton is given by

$$e^\phi = c_1^{-1/4} \left(\frac{3}{2} m \sin \alpha \right)^{-5/6}. \quad (167)$$

This is a static metric. In particular, in the case $q_{\mu\nu}$ is a five-dimensional Minkowski metric $\eta_{\mu\nu}$, the above ten-dimensional metric becomes a warped $\text{AdS}_6 \times S^4$ space [8, 31, 33–35].

It is possible to introduce the KK monopole in the D4-D8 brane system. The ten-dimensional metric is thus written by

$$ds^2 = h^{1/12} \left[h_4^{-3/8} q_{\mu\nu}(X) dx^\mu dx^\nu + h_4^{5/8} (dr^2 + r^2 ds_4^2) \right], \quad (168a)$$

$$ds_4^2 = d\alpha^2 + \frac{1}{4} \cos^2 \alpha [d\Omega_{(2)}^2 + (dv + d\omega)^2], \quad (168b)$$

where $d\omega = \Omega_2$ is the volume form of a unit 2-sphere and h, h_4 are given by (163c). If we perform the T-duality on the ten-dimensional metric (168a), the solution can be viewed as the D5-D7-NS5 brane system;

$$ds^2 = (h_5 h_{\text{NS}})^{3/4} h_7 [(h_5 h_7 h_{\text{NS}})^{-1} \eta_{\mu\nu}(X) dx^\mu dx^\nu + h_7^{-1} \delta_{ij}(Y) dy^i dy^j + h_5^{-1} dv^2 + h_{\text{NS}}^{-1} dz^2], \quad (169a)$$

$$h_5(t, y, z) = c t + \tilde{c} + \sum_{\ell} \frac{M_{\ell}}{[4M_{\text{NS}}|y^i - y_{\ell}^i| + \frac{4M_7}{9}z^3]^{5/3}}, \quad (169b)$$

$$h_7(z) = M_7 z, \quad h_{\text{NS}}(y) = \frac{M_{\text{NS}}}{|y^i - y_0^i|}, \quad (169c)$$

where $\eta_{\mu\nu}(X)$ is a five-dimensional Minkowski metric, and $\delta_{ij}(Y)$ is the three-dimensional Euclidean metric, and M_5, M_7, M_{NS} are constants, and the constants y_{ℓ}^i and y_0^i represent the positions of the branes.

The near horizon structure of the D5-D7-NS5 brane system can be obtained via an appropriate coordinate transformations and embedding of AdS_6 in the IIB theory

$$ds^2 = c_1 \cos^{1/2} \alpha \left[U^2 \eta_{\mu\nu} dx^\mu dx^\nu + c_2 \frac{dU^2}{U^2} + c_3 \left(d\alpha^2 + \frac{1}{4} \cos^2 \alpha d\Omega_{(2)}^2 \right) + c_4 \sin^{2/3} \alpha \cos^{-2} \alpha dv^2 \right], \quad (170)$$

where c_i ($i = 1, \dots, 4$) are constants.

TABLE XVII: Dynamical D5-D7-NS5 brane system. Here \circ denotes the worldvolume coordinate.

Case		0	1	2	3	4	5	6	7	8	9
D5-D7-NS5	D5	\circ	\circ	\circ	\circ	\circ				\circ	
	D7	\circ	\circ	\circ	\circ	\circ	\circ	\circ	\circ		
	NS5	\circ	\circ	\circ	\circ	\circ					\circ
	x^N	t	x^1	x^2	x^3	x^4	y^1	y^2	y^3	v	z

VI. FOUR-DIMENSIONAL COSMOLOGY

In this section, we discuss the application of the time-dependent solutions to study the cosmology. We assume an isotropic and homogeneous three-space in the Friedmann-Robertson-Walker (FRW) universe after compactification. We will not discuss the cases

involving pp-wave because our Universe does not expand when the pp wave is time dependent. We construct the cosmological model from Dp - $D(p+2)$ -NS5 systems. We also provide the brief discussions for other brane systems.

In the following, the p -dimensional background is assumed to be Minkowski spacetime. We also drop the coordinate dependence on X space except for the time coordinate.

Let us consider the case of Dp - $D(p+2)$ -NS5 system to apply the time dependent solution to the cosmological models. We assume that we live in a part of Dp - $D(p+2)$ -NS5 brane. The same branes have to contain three spatial dimensions because four-dimensional universe is isotropic and homogeneous. Then, the ten-dimensional metric in general can be written by

$$ds^2 = -h dt^2 + ds^2(\tilde{X}) + ds^2(Y) + ds^2(W) + ds^2(Z), \quad (171)$$

where

$$ds^2(\tilde{X}) \equiv h \delta_{PQ}(\tilde{X}) d\theta^P d\theta^Q, \quad (172a)$$

$$ds^2(Y) \equiv h_p^{\frac{p+1}{8}}(t, y, z) h_{p+2}^{-\frac{5-p}{8}}(z) h_5(z) \gamma_{ij}(Y) dy^i dy^j, \quad (172b)$$

$$ds^2(W) \equiv h_p^{-\frac{7-p}{8}}(t, y, z) h_{p+2}^{\frac{p+3}{8}}(z) h_5(z) dw^2, \quad (172c)$$

$$ds^2(Z) \equiv h_p^{\frac{p+1}{8}}(t, y, z) h_{p+2}^{\frac{p+3}{8}}(z) u_{ab}(Z) dz^a dz^b, \quad (172d)$$

$$h \equiv h_p^{-\frac{7-p}{8}}(t, y, z) h_{p+2}^{-\frac{5-p}{8}}(z) h_5^{-1/4}(z). \quad (172e)$$

Here, $\delta_{PQ}(\tilde{X})$ is the $(p-1)$ -dimensional Euclidean metric, $\gamma_{ij}(Y)$ and $u_{ab}(Z)$ are the three-, $(6-p)$ -dimensional metrics, respectively and θ^P denotes the coordinate of the $(p-1)$ -dimensional Euclid space \tilde{X} , y^i , z^a are three-, $(6-p)$ -dimensional coordinates. We also assume $h_p = h_p(t, y, z)$, $h_{p+2} = h_{p+2}(z)$ and $h_5 = h_5(z)$.

Now we set $h_p = At + h_1(y, z)$. The ten-dimensional metric (172) can be written as

$$\begin{aligned} ds^2 = & h_{p+2}^{-\frac{5-p}{8}} h_5^{-\frac{1}{4}} \left[1 + \left(\frac{\tau}{\tau_0} \right)^{-\frac{16}{9+p}} h_1 \right]^{-\frac{7-p}{8}} \\ & \times \left[-d\tau^2 + \left(\frac{\tau}{\tau_0} \right)^{\frac{2(7-p)}{9+p}} \left\{ \delta_{PQ}(\tilde{X}) d\theta^P d\theta^Q + h_{p+2} h_5 dw^2 \right\} \right. \\ & \left. + \left\{ 1 + \left(\frac{\tau}{\tau_0} \right)^{-\frac{16}{9+p}} h_1 \right\} \left(\frac{\tau}{\tau_0} \right)^{\frac{2(p+1)}{9+p}} \left\{ h_5 \gamma_{ij}(Y) dy^i dy^j + h_{p+2} u_{ab}(Z) dz^a dz^b \right\} \right], \quad (173) \end{aligned}$$

where the cosmic time τ is defined by

$$\frac{\tau}{\tau_0} = (At)^{\frac{p+9}{16}}, \quad \tau_0 = \frac{16}{(p+9)A}. \quad (174)$$

We discuss the cosmological solution in the lower-dimensional effective theories. We compactify $d(\equiv d_{\tilde{X}} + d_Y + d_W + d_Z)$ dimensions to the $(10 - d)$ -dimensional universe, where $d_{\tilde{X}}$, d_Y , d_W , and d_Z denote the compactified dimensions with respect to the \tilde{X} , Y , W , and Z spaces. The ten-dimensional metric (171) is written by

$$ds^2 = ds^2(M) + ds^2(N), \quad (175)$$

where $ds^2(M)$ is a $(10 - d)$ -dimensional metric and $ds^2(N)$ is a metric of compactified dimensions.

In order to discuss the dynamics of $(10 - d)$ -dimensional universe in the Einstein frame, we use the conformal transformation

$$ds^2(M) = h_p^{B_p} h_{p+2}^{B_{p+2}} h_5^{B_5} ds^2(\bar{M}), \quad (176)$$

where B_p , B_{p+2} and B_5 are expressed as

$$B_p = \frac{-(p+1)d + 8(d_{\tilde{X}} + d_W)}{8(8-d)}, \quad B_{p+2} = \frac{-(p+3)d + 8(d_{\tilde{X}} + d_Y)}{8(8-d)}, \quad (177a)$$

$$B_5 = \frac{-3d + 4(d_Y + d_Z)}{4(8-d)}. \quad (177b)$$

Then, the $(10 - d)$ -dimensional metric in the Einstein frame can be written by

$$ds^2(\bar{M}) = h_p^{B'_p} h_{p+2}^{B'_{p+2}} h_5^{B'_5} \left[-dt^2 + \delta_{P'Q'}(\tilde{X}') d\theta^{P'} d\theta^{Q'} + h_p h_5 \gamma_{k'l'}(Y') dy^{k'} dy^{l'} \right. \\ \left. + h_{p+2} h_5 dw^2 + h_p h_{p+2} u_{a'b'}(Z') dz^{a'} dz^{b'} \right], \quad (178)$$

where B'_p , B'_{p+2} and B'_5 are defined by $B'_p = -B_p - (7 - p)/8$, $B'_{p+2} = -B_{p+2} - (5 - p)/8$, and $B'_5 = B_5 - 1/4$, and \tilde{X}' , Y' , W' , and Z' denote the $(p - 1 - d_{\tilde{X}})$ -, $(3 - d_Y)$ -, $(1 - d_W)$ -, and $(6 - p - d_Z)$ -dimensional spaces, respectively.

If we use $h_p = At + h_1$, the metric (178) can be expressed as

$$ds^2(\bar{M}) = h_{p+2}^{B'_{p+2}} h_5^{B'_5} \left[1 + \left(\frac{\tau}{\tau_0} \right)^{-\frac{2}{B'_{p+2}}} h_1 \right]^{B'_p} \left[-d\tau^2 \right. \\ \left. + \left(\frac{\tau}{\tau_0} \right)^{\frac{2B'_p}{B'_{p+2}}} \left\{ \delta_{P'Q'}(\tilde{X}') d\theta^{P'} d\theta^{Q'} + h_{p+2} h_5 dw^2 \right\} + \left\{ 1 + \left(\frac{\tau}{\tau_0} \right)^{-\frac{2}{B'_{p+2}}} h_1 \right\} \right. \\ \left. \times \left(\frac{\tau}{\tau_0} \right)^{\frac{2(B'_p+1)}{B'_{p+2}}} \left\{ h_5 \gamma_{k'l'}(Y') dy^{k'} dy^{l'} + h_{p+2} u_{a'b'}(Z') dz^{a'} dz^{b'} \right\} \right], \quad (179)$$

where the cosmic time τ is defined by

$$\frac{\tau}{\tau_0} = (At)^{(B'_p+2)/2}, \quad \tau_0 = \frac{2}{(B'_p+2)A}. \quad (180)$$

The solutions we have found do not give an accelerating expansion in our Universe. The scale factor of the $(10-d)$ -dimensional universe is written by $a(\tilde{M}) \propto \tau^{\lambda(\tilde{M})}$, where \tilde{M} is the spatial part of the spacetime M , and $\lambda(\tilde{M})$ is the power exponent of the \tilde{M} space in Jordan frame, and $a_E(\tilde{M}) \propto \tau^{\lambda_E(\tilde{M})}$ denotes the scale factor of the \tilde{M} space in Einstein frame, respectively.

In Table XVIII-XX, we summarize the cosmological solutions derived from the dynamical brane systems which we have discussed in sec.II, III and IV. The expansion law of these brane system is not complicated because the time dependence in the metric comes from only one brane in the intersections.

We present the power exponent of the scale factors of our Universe. These are the similar results with the case of the other partially localized and delocalized intersecting brane solutions. For the solutions (179) involving two intersecting brane in the ten- or eleven-dimensional theories which are related to the supergravity, we have listed the power exponents of the scale factor for the intersection involving two branes in the Tables in [18, 21].

The maximum value of $\lambda(\tilde{M})$ is $3/7$ in the case of D5-D7-NS5 brane system for the ten-dimensional theory, and $2/5$ in the case of M2-, M5-, M2-M2 brane system with KK monopole for the eleven-dimensional model. Although we find the exact time dependent brane solution, the power exponent of our scale factor may be too small to explain our expanding Universe. Furthermore, in order to discuss a de Sitter solution in an intersecting brane system, one may need additional ingredients such as a cosmological constant, which will be discussed in [37]. The power exponents in the four-dimensional Einstein frame depends on how we compactify the internal space in the higher-dimensional theory. We list the power exponent of the fastest expansion of our four-dimensional Universe in the Einstein frame in Table XXI. The fastest expanding case in the Jordan frame has the power $\lambda(\tilde{M}) < 1/2$. Then we cannot obtain any cosmological model which gives a realistic expansion law.

VII. CONCLUSION

In this paper, we have derived the time dependent solutions corresponding to intersecting brane systems. We have obtained the dynamical partially localized intersecting branes in the eleven- or ten-dimensional supergravity models and applied them to the four-dimensional cosmology. Our solutions constructed using the T-duality map between the type IIA and IIB supergravity theories had the similar forms to the known dynamical intersecting brane solutions. These solutions were obtained by replacing a constant c in the warp factor $h = c + h_1$ of a supersymmetric static solution with a linear function of the time. This feature is shared by a wide class of supersymmetric solutions beyond examples considered in this paper. In the case of intersecting branes, the field equations normally indicate that time dependent solutions in supergravity can be found if only one harmonic function in the metric depends on time. It is not easy to find the solutions of the intersecting brane where more than two warp factors depend on both time as well as relative or overall transverse space coordinates.

We then constructed cosmological models from those solutions by smearing some dimensions and compactifying the internal space. Unfortunately, the powers of the scale factors were so small that the solutions could not provide realistic expansions. The solutions in the original higher-dimensional theory imply that as the number p increases the power of the scale factor becomes large. We have found that the intersection with D5-branes in ten-dimensional theory gives the fastest expansion of our Universe because the three-dimensional spatial space of our Universe stays in the transverse space to the D5-brane. Though the power of the scale factor for the transverse space in solutions with the D6-, D7- or D8-branes is larger than those with the D5-brane, the number of the transverse space to these branes is less than three in the partially localized intersecting brane system. Hence, these solutions cannot provide an homogeneous and isotropic universe if we assume that the transverse space to the brane is a part of it.

In the lower-dimensional effective theory, the fastest expansion power is $6/13$, which is found in the case of the D5-D7-NS5 brane for the nine-dimensional effective theory and in the M5-KK monopole system for the nine-dimensional effective theory. This means that we have to include additional matter on the brane in order to obtain a realistic expanding universe such as inflation, matter- or radiation-dominated universe. The properties we

have discovered would also give a clue to investigate cosmological models in more complicated setup, such as more than four intersection of p -brane in the eleven- ten-dimensional supergravity theory.

Although the present examples illustrated in this paper provided neither realistic cosmological model, the features of the solutions or the method to obtain them could open new directions to study how to construct a realistic dynamics of branes as well as an appropriate higher-dimensional cosmological model.

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TABLE XVIII: Dynamical D3-D5-NS5 brane system in ten-dimensional IIB supergravity theory. We compactify $d(\equiv d_{\tilde{X}} + d_Y + d_W + d_Z)$ dimensions to $(10 - d)$ -dimensional Universe, where $d_{\tilde{X}}$, d_Y , d_W , and d_Z denote the compactified dimensions with respect to the \tilde{X} , Y, W, and Z spaces. "TD" shows which brane is time dependent. $\lambda(\tilde{M})$, $\lambda_E(\tilde{M})$ are power exponents of the cosmic time for the scale factor of the space \tilde{M} in Jordan, Einstein frame, respectively.

Branes		0	1	2	3	4	5	6	7	8	9	TD	\tilde{M}	$\lambda(\tilde{M})$	$\lambda_E(\tilde{M})$
D3-D5-NS5	D3	o	o	o				o				\checkmark	$\tilde{X} \& W$	$\lambda(\tilde{X}) = -1/3$	$\lambda_E(\tilde{X}) = \frac{d_Y + d_Z - 4}{12 - 2d_{\tilde{X}} - d_Y - 2d_W - d_Z}$
	D5	o	o	o	o	o								$\lambda(W) = -1/3$	$\lambda_E(W) = \frac{d_Y + d_Z - 4}{12 - 2d_{\tilde{X}} - d_Y - 2d_W - d_Z}$
	NS5	o	o	o					o	o	o			$\lambda(Y) = 1/3$	$\lambda_E(Y) = \frac{4 - d_{\tilde{X}} - d_W}{12 - 2d_{\tilde{X}} - d_Y - 2d_W - d_Z}$
	x^N	t	x^1	x^2	y^1	y^2	y^3	w	z^1	z^2	z^3			$\lambda(Z) = 1/3$	$\lambda_E(Z) = \frac{4 - d_{\tilde{X}} - d_W}{12 - 2d_{\tilde{X}} - d_Y - 2d_W - d_Z}$
D5-D7-NS5	D5	o	o	o	o	o				o		\checkmark	$\tilde{X} \& W$	$\lambda(\tilde{X}) = -1/7$	$\lambda_E(\tilde{X}) = \frac{d_Y + d_Z - 2}{14 - 2d_{\tilde{X}} - d_Y - 2d_W - d_Z}$
	D7	o	o	o	o	o	o	o	o					$\lambda(W) = -1/7$	$\lambda_E(W) = \frac{d_Y + d_Z - 2}{14 - 2d_{\tilde{X}} - d_Y - 2d_W - d_Z}$
	NS5	o	o	o	o	o					o			$\lambda(Y) = 3/7$	$\lambda_E(Y) = \frac{6 - d_{\tilde{X}} - d_W}{14 - 2d_{\tilde{X}} - d_Y - 2d_W - d_Z}$
	x^N	t	x^1	x^2	x^3	x^4	y^1	y^2	y^3	w	z			$\lambda(Z) = 3/7$	$\lambda_E(Z) = \frac{6 - d_{\tilde{X}} - d_W}{14 - 2d_{\tilde{X}} - d_Y - 2d_W - d_Z}$

TABLE XIX: Dynamical M2-brane and KK monopole system in eleven-dimensional supergravity theory. We compactify $d(\equiv d_{\tilde{X}} + d_Y + d_U + d_V + d_Z)$ dimensions to $(10 - d)$ -dimensional Universe, where $d_{\tilde{X}}$, d_Y , d_U , d_V , and d_Z denote the compactified dimensions with respect to the \tilde{X} , Y, U, V, and Z spaces. "TD" denotes which brane (or KK-monopole) is time dependent. $\lambda(\tilde{M})$, $\lambda_E(\tilde{M})$ are power exponents of the cosmic time for the scale factor of the space \tilde{M} in Jordan, Einstein frame, respectively.

Case		0	1	2	3	4	5	6	7	8	9	10	TD	\tilde{M}	$\lambda(\tilde{M})$	$\lambda_E(\tilde{M})$
M2-M2 -KK	M2	o	o	o									\checkmark	$Y \& U$ $\& V \& Z$	$\lambda(Y) = 1/4$	$\lambda_E(Y) = \frac{3 - d_{\tilde{X}}}{12 - 2d_{\tilde{X}} - d_Y - d_U - d_V - d_Z}$
	M2	o			o	o									$\lambda(U) = 1/4$	$\lambda_E(U) = \frac{3 - d_{\tilde{X}}}{12 - 2d_{\tilde{X}} - d_Y - d_U - d_V - d_Z}$
	KK	o	o	o	o	o	o		A_1	A_2	A_3				$\lambda(V) = 1/4$	$\lambda_E(V) = \frac{3 - d_{\tilde{X}}}{12 - 2d_{\tilde{X}} - d_Y - d_U - d_V - d_Z}$
	x^N	t	x^1	x^2	y^1	y^2	u^1	u^2	v	z^1	z^2	z^3			$\lambda(Z) = 1/4$	$\lambda_E(Z) = \frac{3 - d_{\tilde{X}}}{12 - 2d_{\tilde{X}} - d_Y - d_U - d_V - d_Z}$

TABLE XX: Dynamical D2-D6 and KK monopole system in ten-dimensional IIA supergravity theory. We compactify $d(\equiv d_{\tilde{X}} + d_Y + d_V + d_Z)$ dimensions to $(10 - d)$ -dimensional Universe, where $d_{\tilde{X}}$, d_Y , d_V , and d_Z denote the compactified dimensions with respect to the \tilde{X} , Y, V, and Z spaces. "TD" shows which brane (or KK-monopole) is time dependent. $\lambda(\tilde{M})$, $\lambda_E(\tilde{M})$ are power exponents of the cosmic time for the scale factor of the space \tilde{M} in Jordan, Einstein frame, respectively.

Branes		0	1	2	3	4	5	6	7	8	9	TD	\tilde{M}	$\lambda(\tilde{M})$	$\lambda_E(\tilde{M})$
D2-D6-KK	D2	o	o	o								\checkmark	$Y \& V \& Z$	$\lambda(Y) = 3/11$	$\lambda_E(Y) = \frac{3 - d_{\tilde{X}}}{11 - 2d_{\tilde{X}} - d_Y - d_V - d_Z}$
	D6	o	o	o	o	o	o	o						$\lambda(V) = 3/11$	$\lambda_E(V) = \frac{3 - d_{\tilde{X}}}{11 - 2d_{\tilde{X}} - d_Y - d_V - d_Z}$
	KK	o	o	o	A_1	A_2	A_3		o	o	o			$\lambda(Z) = 3/11$	$\lambda_E(Z) = \frac{3 - d_{\tilde{X}}}{11 - 2d_{\tilde{X}} - d_Y - d_V - d_Z}$
	x^N	t	x^1	x^2	y^1	y^2	y^3	v	z^1	z^2	z^3				

TABLE XXI: The power exponent of the fastest expansion in the Einstein frame for dynamical brane and KK monopole in ten- or eleven-dimensional supergravity theory. “TD” in the table denotes which brane is time dependent.

Branes	TD	dim(M)	\bar{M}	d	$\lambda_E(\bar{M})$
D3-D5-NS5	D3	7	$\tilde{X} \text{ \& } Y \text{ \& } W \text{ \& } Z$	$(d_{\tilde{X}}, d_Y, d_W, d_Z) = (0, 1, 0, 2)$	4/9
	D3	7	$\tilde{\tilde{X}} \text{ \& } Y \text{ \& } W \text{ \& } Z$	$(d_{\tilde{\tilde{X}}}, d_Y, d_W, d_Z) = (0, 2, 0, 1)$	4/9
D5-D7-NS5	D5	9	$\tilde{X} \text{ \& } Y \text{ \& } W \text{ \& } Z$	$(d_{\tilde{X}}, d_Y, d_W, d_Z) = (0, 1, 0, 0)$	6/13
M2-M2-KK	M2	6	$\tilde{X} \text{ \& } Y \text{ \& } U \text{ \& } V \text{ \& } Z$	$(d_{\tilde{X}}, d_Y, d_U, d_V, d_Z) = (0, 1, 2, 0, 2)$	3/7
D2-D6-KK	D2	6	$\tilde{X} \text{ \& } Y \text{ \& } V \text{ \& } U$	$(d_{\tilde{X}}, d_Y, d_V, d_Z) = (0, 2, 0, 2)$	3/7